

## Physics

1. The Kirchoff's first law ( $\sum i = 0$ ) and second law ( $\sum iR = \sum E$ ), where the symbols have their usual meanings, are respectively based on :
  - (a) conservation of charge, conservation of momentum
  - (b) conservation of energy, conservation of charge
  - (c) conservation of momentum, conservation of charge
  - (d) conservation of charge, conservation of energy
2. Needles  $N_1, N_2$  and  $N_3$  are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will :
  - (a) attract  $N_1$  and  $N_2$  strongly but repel  $N_3$
  - (b) attract  $N_1$  strongly,  $N_2$  weakly and repel  $N_3$  weakly
  - (c) attract  $N_1$  strongly, but repel  $N_2$  and  $N_3$  weakly
  - (d) attract all three of them
3. A material 'B' has twice the specific resistance of 'A'. A circular wire made of 'B' has twice the diameter of a wire made of 'A'. Then for the two wires to have the same resistance, the ratio  $l_B / l_A$  of their respective lengths must be :
 

(a) 1	(b) $\frac{1}{2}$
(c) $\frac{1}{4}$	(d) 2
4. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a :
 

(a) helix	(b) straight line
(c) ellipse	(d) circle
5. An electric dipole is placed at an angle of  $30^\circ$  to a non-uniform electric field. The dipole will experience :
  - (a) a translational force only in the direction of the field
  - (b) a translational force only in a direction normal to the direction of the field
  - (c) a torque as well as a translational force
  - (d) a torque only
6. A particle located at  $x = 0$  at time  $t = 0$ , starts moving along the positive  $x$ -direction with a velocity ' $v$ ' that varies as  $v = \alpha\sqrt{x}$ . The displacement of the particle varies with time as :
 

(a) $t^2$	(b) $t$
(c) $t^{1/2}$	(d) $t^3$
7. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is  $4 \text{ ms}^{-1}$ . The kinetic energy of the other mass is :
 

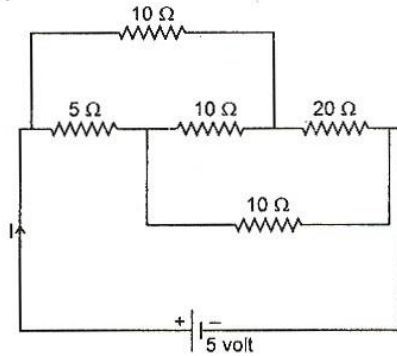
(a) 144 J	(b) 288 J
(c) 192 J	(d) 96 J
8. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed  $v \text{ ms}^{-1}$ . The velocity of sound in air is  $300 \text{ ms}^{-1}$ . If the person can hear frequencies upto a maximum of 10,000 Hz, the maximum value of  $v$  upto which he can hear the whistle is :
 

(a) $15\sqrt{2} \text{ ms}^{-1}$	(b) $15/\sqrt{2} \text{ ms}^{-1}$
(c) $15 \text{ ms}^{-1}$	(d) $30 \text{ ms}^{-1}$
9. A mass of  $M$  kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of  $45^\circ$  with the initial vertical direction is :
 

(a) $Mg(\sqrt{2} + 1)$	(b) $Mg\sqrt{2}$
(c) $\frac{Mg}{\sqrt{2}}$	(d) $Mg(\sqrt{2} - 1)$

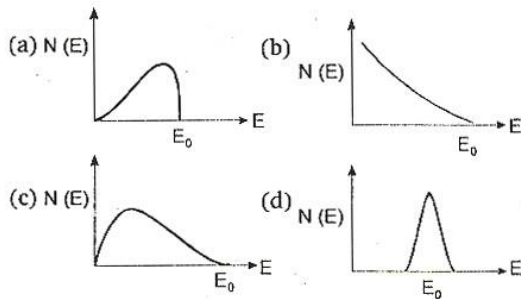
10. A particle of mass 100 g is thrown vertically upwards with a speed of 5 m/s. The work done by the force of gravity during the time the particle goes up is :  
 (a) -0.5 J (b) -1.25 J  
 (c) 1.25 J (d) 0.5 J
11. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is :  
 (a) 0.01 s (b) 10 s  
 (c) 0.1 s (d) 100 s
12. Starting from the origin a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy ?  
 (a)  $\frac{1}{6}$  s (b)  $\frac{1}{4}$  s  
 (c)  $\frac{1}{3}$  s (d)  $\frac{1}{12}$  s
13. Assuming the sun to be a spherical body of radius  $R$  at a temperature of  $T$  K, evaluate the total radiant power, incident on earth, at a distance  $r$  from the sun :  
 (a)  $4\pi r_0^2 R^2 \sigma T^4 / r^2$  (b)  $\pi r_0^2 R^2 \sigma T^4 / r^2$   
 (c)  $r_0^2 R^2 \sigma T^4 / 4\pi r^2$  (d)  $R^2 \sigma T^4 / r^2$   
 where  $r_0$  is the radius of the earth and  $\sigma$  is Stefan's constant.
14. Which of the following units denotes the dimensions  $[ML^2/Q^2]$ , where  $Q$  denotes the electric charge ?  
 (a)  $Wb/m^2$  (b) henry (H)  
 (c)  $H/m^2$  (d) weber (Wb)
15. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider  $g = 10 \text{ m/s}^2$ :  
 (a) 4 N (b) 16 N  
 (c) 20 N (d) 22 N
16. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is :  
 (a) 105 Hz (b) 1.05 Hz  
 (c) 1050 Hz (d) 10.5 Hz
17. Consider a two particle system with particles having masses  $m_1$  and  $m_2$ . If the first particle is pushed towards the centre of mass through a distance  $d$ , by what distance should the second particle be moved, so as to keep the centre of mass at the same position ?  
 (a)  $\frac{m_2}{m_1} d$  (b)  $\frac{m_1}{m_1 + m_2} d$   
 (c)  $\frac{m_1}{m_2} d$  (d)  $d$
18. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1s, the force of the blow exerted by the ball on the hand of the player is equal to :  
 (a) 150 N (b) 3 N  
 (c) 30 N (d) 300 N
19. In a common-base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor ( $\beta$ ) will be :  
 (a) 49 (b) 50  
 (c) 51 (d) 48
20. A thermocouple is made from two metals, Antimony and Bismuth. If one junction of the couple is kept hot and the other is kept cold, then, an electric current will :  
 (a) flow from Antimony to Bismuth at the hot junction  
 (b) flow from Bismuth to Antimony at the cold junction  
 (c) not flow through the thermocouple  
 (d) flow from Antimony to Bismuth at the cold junction
21. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5V. The incident radiation lies in :  
 (a) ultra-violet region  
 (b) infra-red region  
 (c) visible region  
 (d) X-ray region
22. An alpha nucleus of energy  $\frac{1}{2}mv^2$  bombards a heavy nuclear target of charge  $Ze$ . Then the distance of closest approach for the alpha nucleus will be proportional to :  
 (a)  $v^2$  (b)  $1/m$   
 (c)  $1/v^4$  (d)  $1/Ze$
23. The time taken by a photoelectron to come out after the photon strikes is approximately :  
 (a)  $10^{-4}$  s (b)  $10^{-10}$  s  
 (c)  $10^{-16}$  s (d)  $10^{-1}$  s
24. When  ${}_3\text{Li}^7$  nuclei are bombarded by protons, and the resultant nuclei are  ${}_4\text{Be}^8$ , the emitted particles will be :  
 (a) alpha particles (b) beta particles  
 (c) gamma photons (d) neutrons

25. The current  $I$  drawn from the 5 volt source will be :



- (a) 0.33 A                      (b) 0.5 A  
(c) 0.67 A                      (d) 0.17 A

26. The energy spectrum of  $\beta$ -particles [number  $N(E)$  as a function of  $\beta$ -energy  $E$ ] emitted from a radioactive source is :



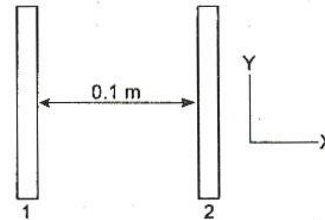
27. In a series resonant  $LCR$  circuit, the voltage across  $R$  is 100 volts and  $R = 1 \text{ k}\Omega$  with  $C = 2 \mu\text{F}$ . The resonant frequency  $\omega$  is 200 rad/s. At resonance the voltage across  $L$  is :
- (a)  $2.5 \times 10^{-2} \text{ V}$                       (b) 40 V  
(c) 250 V                                      (d)  $4 \times 10^{-3} \text{ V}$

28. The resistance of a bulb filament is  $100 \Omega$  at a temperature of  $100^\circ\text{C}$ . If its temperature coefficient of resistance be 0.005 per  $^\circ\text{C}$ , its resistance will become  $200 \Omega$  at a temperature of :
- (a)  $300^\circ\text{C}$                                       (b)  $400^\circ\text{C}$   
(c)  $500^\circ\text{C}$                                       (d)  $200^\circ\text{C}$

29. Two insulating plates are both uniformly charged in such a way that the potential difference between them is  $V_2 - V_1 = 20 \text{ V}$ . (i.e., plate 2 is at a higher potential). The plates are separated by  $d = 0.1 \text{ m}$  and can be treated as

infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2 ?

( $e = 1.6 \times 10^{-19} \text{ C}$ ,  $m_0 = 9.11 \times 10^{-31} \text{ kg}$ )



- (a)  $2.65 \times 10^6 \text{ m/s}$                       (b)  $7.02 \times 10^{12} \text{ m/s}$   
(c)  $1.87 \times 10^6 \text{ m/s}$                       (d)  $32 \times 10^{-19} \text{ m/s}$

30. In an AC generator, a coil with  $N$  turns, all of the same area  $A$  and total resistance  $R$ , rotates with frequency  $\omega$  in a magnetic field  $B$ . The maximum value of emf generated in the coil is :
- (a)  $N.A.B.R. \omega$                                       (b)  $N.A.B$   
(c)  $N.A.B.R$                                       (d)  $N.A.B. \omega$

31. A solid which is not transparent to visible light and whose conductivity increases with temperature is formed by :
- (a) ionic binding  
(b) covalent binding  
(c) Van der Waal's binding  
(d) metallic binding

32. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let  $D_1$  and  $D_2$  be angles of minimum deviation for red and blue light respectively in a prism of this glass. then, :
- (a)  $D_1 < D_2$   
(b)  $D_1 = D_2$   
(c)  $D_1$  can be less than or greater than  $D_2$  depending upon the angle of prism  
(d)  $D_1 > D_2$

33. If the ratio of the concentration of electrons to that of holes in a semiconductor is  $\frac{7}{5}$  and the ratio of currents is  $\frac{7}{4}$ , then what is the ratio of their drift velocities ?
- (a)  $\frac{5}{8}$     (b)  $\frac{4}{5}$   
(c)  $\frac{5}{4}$     (d)  $\frac{4}{7}$

34. In a Wheatstone's bridge, three resistances  $P$ ,  $Q$  and  $R$  are connected in the three arms and the fourth arm is formed by two resistances  $S_1$  and  $S_2$  connected in parallel. The condition for the bridge to be balanced will be :



(a)  $\frac{P}{Q} = \frac{2R}{S_1 + S_2}$       (b)  $\frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$   
 (c)  $\frac{P}{Q} = \frac{R(S_1 + S_2)}{2S_1 S_2}$       (d)  $\frac{P}{Q} = \frac{R}{S_1 + S_2}$

35. The flux linked with a coil at any instant 't' is given by :

$$\phi = 10t^2 - 50t + 250$$

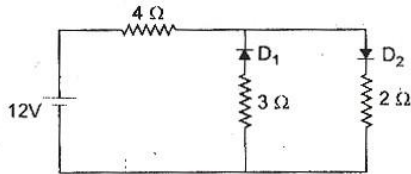
The induced emf at  $t = 3$  s is :

- (a) -190 V      (b) -10 V  
 (c) 10 V      (d) 190 V

36. A long solenoid has 200 turns per cm and carries a current  $i$ . The magnetic field at its centre is  $6.28 \times 10^{-2}$  weber/m<sup>2</sup>. Another long solenoid has 100 turns per cm and it carries a current  $i/3$ . The value of the magnetic field at its centre is :

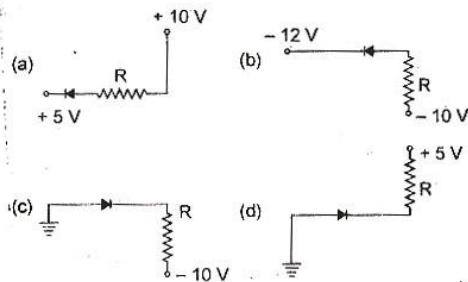
- (a)  $1.05 \times 10^{-2}$  weber/m<sup>2</sup>  
 (b)  $1.05 \times 10^{-5}$  weber/m<sup>2</sup>  
 (c)  $1.05 \times 10^{-3}$  weber/m<sup>2</sup>  
 (d)  $1.05 \times 10^{-4}$  weber/m<sup>2</sup>

37. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit ?

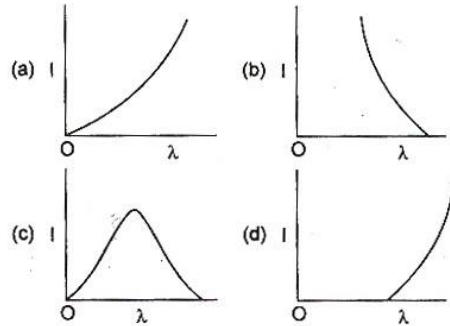


- (a) 1.71 A      (b) 2.00 A  
 (c) 2.31 A      (d) 1.33 A

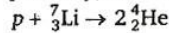
38. In the following, which one of the diodes is reverse biased ?



The anode voltage of a photocell is kept fixed. The wavelength  $\lambda$  of the light falling on the cathode is gradually changed. The plate current  $I$  of the photocell varies as follows :



40. If the binding energy per nucleon in  ${}^7_3\text{Li}$  and  ${}^4_2\text{He}$  nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction :



energy of proton must be :

- (a) 28.24 MeV      (b) 17.28 MeV  
 (c) 1.46 MeV      (d) 39.2 MeV

41. An electric bulb is rated 220 volt - 100 watt. The power consumed by it when operated on 110 volt will be :

- (a) 75 watt      (b) 40 watt  
 (c) 25 watt      (d) 50 watt

42. The 'rad' is the correct unit used to report the measurement of :

- (a) the ability of a beam of gamma ray photons to produce ions in a target  
 (b) the energy delivered by radiation to a target  
 (c) the biological effect of radiation  
 (d) the rate of decay of a radioactive source

43. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency  $\omega$ . The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time :

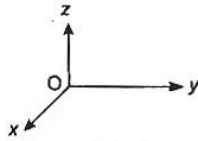
- (a) at the mean position of the platform  
 (b) for an amplitude of  $g/\omega^2$   
 (c) for an amplitude of  $g^2/\omega^2$   
 (d) at the highest position of the platform

44. Four point masses, each of value  $m$ , are placed at the corners of a square ABCD of side  $l$ . The moment of inertia of this system about an axis passing through A and parallel to BD is :

- (a)  $2ml^2$       (b)  $\sqrt{3}ml^2$   
 (c)  $3ml^2$       (d)  $ml^2$

45. A force of  $-F \hat{k}$  acts on O, the origin of the co-ordinate system. The torque about the point  $(1, -1)$  is :

- (a)  $F(\hat{i} - \hat{j})$   
 (b)  $-F(\hat{i} + \hat{j})$   
 (c)  $F(\hat{i} + \hat{j})$   
 (d)  $-F(\hat{i} - \hat{j})$



46. The potential energy of a 1 kg particle free to move along the x-axis is given by

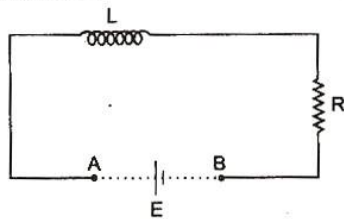
$$V(x) = \left( \frac{x^4}{4} - \frac{x^2}{2} \right) \text{ J.}$$

The total mechanical energy of the particle is 2 J. Then, the maximum speed (in m/s) is :

- (a)  $3/\sqrt{2}$  (b)  $\sqrt{2}$   
 (c)  $1/\sqrt{2}$  (d) 2
47. A thin circular ring of mass  $m$  and radius  $R$  is rotating about its axis with a constant angular velocity  $\omega$ . Two objects each of mass  $M$  are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity  $\omega' =$  :
- (a)  $\frac{\omega(m+2M)}{m}$  (b)  $\frac{\omega(m-2M)}{(m+2M)}$   
 (c)  $\frac{\omega m}{(m+M)}$  (d)  $\frac{\omega m}{(m+2M)}$

48. A wire elongates by  $l$  mm when a load  $W$  is hanged from it. If the wire goes over a pulley and two weights  $W$  each are hung at the two ends, the elongation of the wire will be (in mm) :
- (a)  $l$  (b)  $2l$   
 (c) zero (d)  $l/2$

49. An inductor ( $L = 100 \text{ mH}$ ), a resistor ( $R = 100 \Omega$ ) and a battery ( $E = 100 \text{ V}$ ) are initially connected in series as shown in the figure. After a long time the battery is disconnected after short circuiting the points A and B. The current in the circuit 1 ms after the short circuit is :



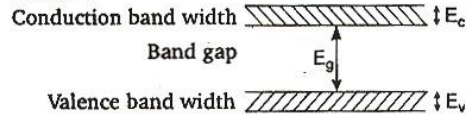
- (a)  $1/e \text{ A}$  (b)  $e \text{ A}$   
 (c)  $0.1 \text{ A}$  (d)  $1 \text{ A}$
50. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire then in equilibrium condition, the ratio of the

magnitude of the electric fields at the surfaces of spheres A and B is :

- (a) 4 : 1 (b) 1 : 2  
 (c) 2 : 1 (d) 1 : 4
51. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature  $T_0$ , while box B contains one mole of helium at temperature  $(7/3) T_0$ . The boxes are then put into thermal contact with each other, and heat flows between them until the gases reach a common final temperature (Ignore the heat capacity of boxes). Then, the final temperature of the gases,  $T_f$ , in terms of  $T_0$  is :
- (a)  $T_f = \frac{3}{7} T_0$  (b)  $T_f = \frac{7}{3} T_0$   
 (c)  $T_f = \frac{3}{2} T_0$  (d)  $T_f = \frac{5}{2} T_0$

52. The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process the temperature of the gas increases by  $7^\circ\text{C}$ . The gas is :  
 ( $R = 8.3 \text{ J mol}^{-1} \text{ K}^{-1}$ )
- (a) diatomic  
 (b) triatomic  
 (c) a mixture of monoatomic and diatomic  
 (d) monoatomic

53. The lattice constant of this semiconductor is decreased, then which of the following is correct ?



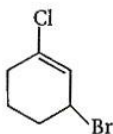
- (a) All  $E_c, E_g, E_v$  increase  
 (b)  $E_c$  and  $E_v$  increase, but  $E_g$  decreases  
 (c)  $E_c$  and  $E_v$  decrease, but  $E_g$  increases  
 (d) All  $E_c, E_g, E_v$  decrease
54. The rms value of the electric field of the light coming from the sun is 720 N/C. The average total energy density of the electromagnetic wave is :
- (a)  $4.58 \times 10^{-6} \text{ J/m}^3$  (b)  $6.37 \times 10^{-9} \text{ J/m}^3$   
 (c)  $81.35 \times 10^{-12} \text{ J/m}^3$  (d)  $3.3 \times 10^{-3} \text{ J/m}^3$

55. If the terminal speed of a sphere of gold (density =  $19.5 \text{ kg/m}^3$ ) is 0.2 m/s in a viscous liquid (density =  $1.5 \text{ kg/m}^3$ ), find the terminal speed of a sphere of silver (density =  $10.5 \text{ kg/m}^3$ ) of the same size in the same liquid.
- (a) 0.4 m/s (b) 0.133 m/s  
 (c) 0.1 m/s (d) 0.2 m/s

## Chemistry

56. HBr reacts with  $\text{CH}_2=\text{CH}-\text{OCH}_3$  under anhydrous conditions at room temperature to give :
- $\text{CH}_3\text{CHO}$  and  $\text{CH}_3\text{Br}$
  - $\text{BrCH}_2\text{CHO}$  and  $\text{CH}_3\text{OH}$
  - $\text{BrCH}_2-\text{CH}_2-\text{OCH}_3$
  - $\text{H}_3\text{C}-\text{CHBr}-\text{OCH}_3$

57. The IUPAC name of the compound shown below is :



- 2-bromo-6-chlorocyclohex-1-ene
  - 6-bromo-2-chlorocyclohexene
  - 3-bromo-1-chlorocyclohexene
  - 1-bromo-3-chlorocyclohexene
58. The increasing order of the rate of HCN addition to compounds A-D is :
- HCHO
  - $\text{CH}_3\text{COCH}_3$
  - $\text{PhCOCH}_3$
  - $\text{PhCOPh}$
- $A < B < C < D$
  - $D < B < C < A$
  - $D < C < B < A$
  - $C < D < B < A$
59. How many moles of magnesium phosphate,  $\text{Mg}_3(\text{PO}_4)_2$  will contain 0.25 mole of oxygen atoms ?
- 0.02
  - $3.125 \times 10^{-2}$
  - $1.25 \times 10^{-2}$
  - $2.5 \times 10^{-2}$
60. According to Bohr's theory, the angular momentum of an electron in 5th orbit is :
- $25 \frac{h}{\pi}$
  - $1.0 \frac{h}{\pi}$
  - $10 \frac{h}{\pi}$
  - $2.5 \frac{h}{\pi}$
61. Which of the following molecules/ions does not contain unpaired electrons ?
- $\text{O}_2^{2-}$
  - $\text{B}_2$
  - $\text{N}_2^+$
  - $\text{O}_2$
62. Total volume of atoms present in a face-centred cubic unit cell of a metal is ( $r$  is atomic radius) :
- $\frac{20}{3} \pi r^3$
  - $\frac{24}{3} \pi r^3$
  - $\frac{12}{3} \pi r^3$
  - $\frac{16}{3} \pi r^3$

63. A reaction was found to be second order with respect to the concentration of carbon monoxide. If the concentration of carbon monoxide is doubled, with everything else kept the same, the rate of reaction will :

- remain unchanged
- triple
- increases by a factor of 4
- double

64. Which of the following chemical reactions depicts the oxidizing behaviour of  $\text{H}_2\text{SO}_4$  ?

- $2\text{HI} + \text{H}_2\text{SO}_4 \rightarrow \text{I}_2 + \text{SO}_2 + 2\text{H}_2\text{O}$
- $\text{Ca}(\text{OH})_2 + \text{H}_2\text{SO}_4 \rightarrow \text{CaSO}_4 + 2\text{H}_2\text{O}$
- $\text{NaCl} + \text{H}_2\text{SO}_4 \rightarrow \text{NaHSO}_4 + \text{HCl}$
- $2\text{PCl}_5 + \text{H}_2\text{SO}_4 \rightarrow 2\text{POCl}_3 + 2\text{HCl} + \text{SO}_2\text{Cl}_2$

65. The IUPAC name for the complex  $[\text{Co}(\text{NO}_2)(\text{NH}_3)_5]\text{Cl}_2$  is :

- nitrito-N-pentamminecobalt (III) chloride
- nitrito-N-pentamminecobalt (II) chloride
- pentammine nitrito-N-cobalt (II) chloride
- pentaamine nitrito-N-cobalt (III) chloride

66. The term anomers of glucose refers to :

- isomers of glucose that differ in configurations at carbons one and four (C-1 and C-4)
- a mixture of (D)-glucose and (L)-glucose
- enantiomers of glucose
- isomers of glucose that differ in configuration at carbon one (C-1)

67. In the transformation of  ${}_{92}^{238}\text{U}$  to  ${}_{92}^{234}\text{U}$ , if one emission is an  $\alpha$ -particle, what should be the other emission (s) ?

- Two  $\beta^-$
- Two  $\beta^-$  and one  $\beta^+$
- One  $\beta^-$  and one  $\gamma$
- One  $\beta^+$  and one  $\beta^-$

68. Phenyl magnesium bromide reacts with methanol to give :

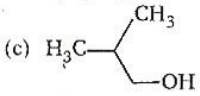
- a mixture of anisole and  $\text{Mg}(\text{OH})\text{Br}$
- a mixture of benzene and  $\text{Mg}(\text{OMe})\text{Br}$
- a mixture of toluene and  $\text{Mg}(\text{OH})\text{Br}$
- a mixture of phenol and  $\text{Mg}(\text{Me})\text{Br}$

69.  $\text{CH}_3\text{Br} + \text{Nu}^- \rightarrow \text{CH}_3-\text{Nu} + \text{Br}^-$

The decreasing order of the rate of the above reaction with nucleophiles ( $\text{Nu}^-$ ) A to D is :

$[\text{Nu}^- = (\text{A})\text{PhO}^-, (\text{B})\text{AcO}^-, (\text{C})\text{HO}^-, (\text{D})\text{CH}_3\text{O}^-]$



- (a)  $D > C > A < B$     (b)  $D > C > B > A$   
 (c)  $A > B > C > D$     (d)  $B > D > C > A$
70. The pyrimidine bases present in DNA are :  
 (a) cytosine and adenine  
 (b) cytosine and guanine  
 (c) cytosine and thymine  
 (d) cytosine and uracil
71. Among the following the one that gives positive iodoform test upon reaction with  $I_2$  and NaOH is :  
 (a)  $CH_3CH_2CH(OH)CH_2CH_3$   
 (b)  $C_6H_5CH_2CH_2OH$   
 (c)   
 (d)  $PhCHOHCH_3$
72. The increasing order of stability of the following free radicals is :  
 (a)  $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C}$   
 (b)  $(C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$   
 (c)  $(C_6H_5)_2\dot{C}H < (C_6H_5)_3\dot{C} < (CH_3)_3\dot{C} < (CH_3)_2\dot{C}H$   
 (d)  $(CH_3)_2\dot{C}H < (CH_3)_3\dot{C} < (C_6H_5)_3\dot{C} < (C_6H_5)_2\dot{C}H$
73. Uncertainty in the position of an electron (mass =  $9.1 \times 10^{-31}$  kg) moving with a velocity  $300 \text{ ms}^{-1}$ , accurate upon 0.001% will be :  
 (a)  $19.2 \times 10^{-2} \text{ m}$     (b)  $5.76 \times 10^{-2} \text{ m}$   
 (c)  $1.92 \times 10^{-2} \text{ m}$     (d)  $3.84 \times 10^{-2} \text{ m}$   
 ( $h = 6.63 \times 10^{-34} \text{ Js}$ )
74. Phosphorus pentachloride dissociates as follows, in a closed reaction vessel,  
 $PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$   
 If total pressure at equilibrium of the reaction mixture is  $P$  and degree of dissociation of  $PCl_5$  is  $x$ , the partial pressure of  $PCl_3$  will be :  
 (a)  $\left(\frac{x}{x+1}\right)P$     (b)  $\left(\frac{2x}{1-x}\right)P$   
 (c)  $\left(\frac{x}{x-1}\right)P$     (d)  $\left(\frac{x}{1-x}\right)P$
75. The standard enthalpy of formation ( $\Delta H_f^\circ$ ) at 298 K for methane,  $CH_4(g)$ , is  $-74.8 \text{ kJ mol}^{-1}$ . The additional information required to determine the average energy for C—H bond formation would be :  
 (a) the dissociation energy of  $H_2$  and enthalpy of sublimation of carbon  
 (b) latent heat of vaporization of methane  
 (c) the first four ionization energies of carbon and electron gain enthalpy of hydrogen  
 (d) the dissociation energy of hydrogen molecule,  $H_2$
76. Among the following mixtures, dipole-dipole as the major interaction, is present in :  
 (a) benzene and ethanol  
 (b) acetonitrile and acetone  
 (c) KCl and water  
 (d) benzene and carbon tetrachloride
77. Fluorobenzene ( $C_6H_5F$ ) can be synthesized in the laboratory :  
 (a) by heating phenol with HF and KF  
 (b) from aniline by diazotisation followed by heating the diazonium salt with  $HFBF_4$   
 (c) by direct fluorination of benzene with  $F_2$  gas  
 (d) by reacting bromobenzene with NaF solution
78. A metal,  $M$  forms chlorides in its +2 and +4 oxidation states. Which of the following statements about these chlorides is correct ?  
 (a)  $MCl_2$  is more volatile than  $MCl_4$   
 (b)  $MCl_2$  is more soluble in anhydrous ethanol than  $MCl_4$   
 (c)  $MCl_2$  is more ionic than  $MCl_4$   
 (d)  $MCl_2$  is more easily hydrolysed than  $MCl_4$
79. Which of the following statement is true ?  
 (a)  $H_3PO_3$  is a stronger acid than  $H_2SO_3$   
 (b) In aqueous medium HF is a stronger acid than HCl  
 (c)  $HClO_4$  is a weaker acid than  $HClO_3$   
 (d)  $HNO_3$  is a stronger acid than  $HNO_2$
80. The molar conductivities  $\wedge_{NaOAc}^\circ$  and  $\wedge_{HCl}^\circ$  at infinite dilution in water at  $25^\circ\text{C}$  are 91.0 and  $426.2 \text{ S cm}^2/\text{mol}$  respectively. To calculate  $\wedge_{HOAc}^\circ$ , the additional value required is :  
 (a)  $\wedge_{H_2O}^\circ$     (b)  $\wedge_{KCl}^\circ$   
 (c)  $\wedge_{NaOH}^\circ$     (d)  $\wedge_{NaCl}^\circ$
81. Which one of the following sets of ions represents a collection of isoelectronic species ?  
 (a)  $K^+$ ,  $Cl^-$ ,  $Ca^{2+}$ ,  $Sc^{3+}$   
 (b)  $Ba^{2+}$ ,  $Sr^{2+}$ ,  $K^+$ ,  $S^{2-}$   
 (c)  $N^{3-}$ ,  $O^{2-}$ ,  $F^-$ ,  $S^{2-}$   
 (d)  $Li^+$ ,  $Na^+$ ,  $Mg^{2+}$ ,  $Ca^{2+}$

82. The correct order of increasing acid strength of the compounds :



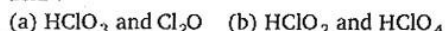
is :



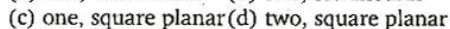
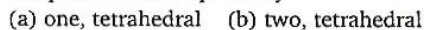
83. In which of the following molecules/ions all the bonds are not equal ?



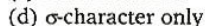
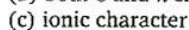
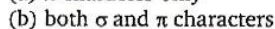
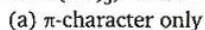
84. What products are expected from the disproportionation reaction of hypochlorous acid ?



85. Nickel ( $Z = 28$ ) combines with a uninegative monodentate ligand  $X^-$  to form a paramagnetic complex  $[\text{NiX}_4]^{2-}$ . The number of unpaired electron(s) in the nickel and geometry of this complex ion are respectively :



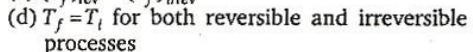
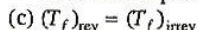
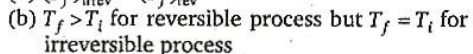
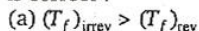
86. In  $\text{Fe}(\text{CO})_5$ , the Fe—C bond possesses :



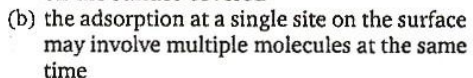
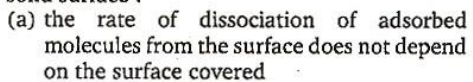
87. The increasing order of the first ionization enthalpies of the elements B, P, S and F (lowest first) is :



88. An ideal gas is allowed to expand both reversibly and irreversibly in an isolated system. If  $T_i$  is the initial temperature and  $T_f$  is the final temperature, which of the following statements is correct ?



89. In Langmuir's model of adsorption of a gas on a solid surface :



(c) the mass of gas striking a given area of surface is proportional to the pressure of the gas

(d) the mass of gas striking a given area of surface is independent of the pressure of the gas

90. Rate of a reaction can be expressed by Arrhenius equation as :

$$k = Ae^{-E/RT}$$

In this equation,  $E$  represents :

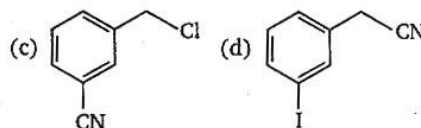
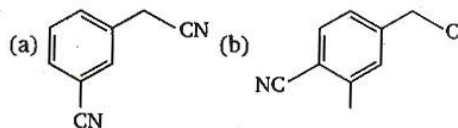
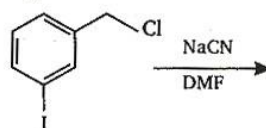
(a) the energy above which all the colliding molecules will react

(b) the energy below which colliding molecules will not react

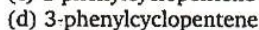
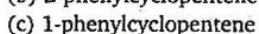
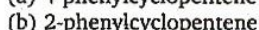
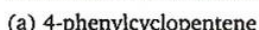
(c) the total energy of the reacting molecules at a temperature,  $T$

(d) the fraction of molecules with energy greater than the activation energy of the reaction

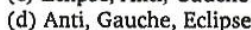
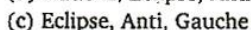
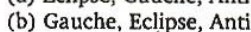
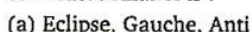
91. The structure of the major product formed in the following reaction is :



92. Reaction of *trans*-2-phenyl-1-bromocyclopentane on reaction with alcoholic KOH produces :



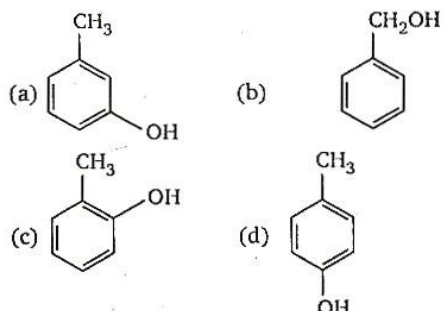
93. Increasing order of stability among the three main conformations (*i.e.*, Eclipse, Anti, Gauche) of 2-fluoroethanol is :



94. The structure of the compound that gives a

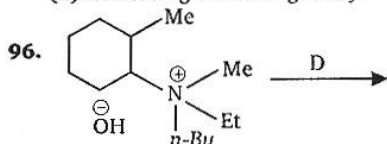


tribromo derivative on treatment with bromine water is :

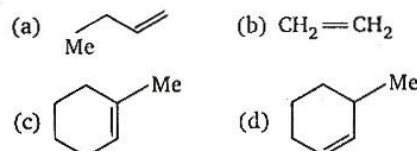


95. The decreasing values of bond angles from  $\text{NH}_3$  ( $107^\circ$ ) to  $\text{SbH}_3$  ( $91^\circ$ ) down group-15 of the periodic table is due to :

- (a) increasing bp-bp repulsion  
 (b) increasing  $p$ -orbital character in  $sp^3$   
 (c) decreasing lp-bp repulsion  
 (d) decreasing electronegativity



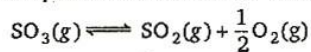
The alkene formed as a major product in the above elimination reaction is :



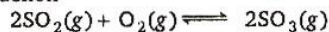
97. The 'spin-only' magnetic moment [in units of Bohr magneton, ( $\mu_B$ )] of  $\text{Ni}^{2+}$  in aqueous solution would be (Atomic number of Ni = 28) :

- (a) 2.84 (b) 4.90  
 (c) 0 (d) 1.73

98. The equilibrium constant for the reaction



is  $K_c = 4.9 \times 10^{-2}$ . The value of  $K_c$  for the reaction



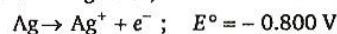
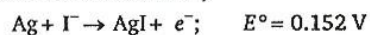
will be :

- (a) 416 (b)  $2.40 \times 10^{-3}$   
 (c)  $9.8 \times 10^{-2}$  (d)  $4.9 \times 10^{-2}$

99. Following statements regarding the periodic trends of chemical reactivity of the alkali metals and the halogens are given. Which of these statements gives the correct picture ?

- (a) The reactivity decreases in the alkali metals but increases in the halogens with increase in atomic number down the group  
 (b) In both the alkali metals and the halogens the chemical reactivity decreases with increase in atomic number down the group  
 (c) Chemical reactivity increases with increase in atomic number down the group in both the alkali metals and halogens  
 (d) In alkali metals the reactivity increases but in the halogens it decreases with increase in atomic number down the group

100. Given the data at  $25^\circ\text{C}$ ,

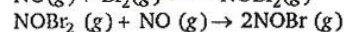
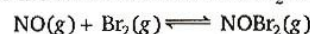


What is the value of  $\log K_{sp}$  for AgI ?

$$\left( 2.303 \frac{RT}{F} = 0.059 \text{ V} \right)$$

- (a) - 8.12 (b) + 8.612  
 (c) - 37.83 (d) - 16.13

101. The following mechanism has been proposed for the reaction of NO with  $\text{Br}_2$  to form NOBr :



If the second step is the rate determining step, the order of the reaction with respect to NO (g) is :

- (a) 1 (b) 0  
 (c) 3 (d) 2

102. Lanthanoid contraction is caused due to :

- (a) the appreciable shielding on outer electrons by  $4f$  electrons from the nuclear charge  
 (b) the appreciable shielding on outer electrons by  $5d$  electrons from the nuclear charge  
 (c) the same effective nuclear charge from Ce to Lu  
 (d) the imperfect shielding on outer electrons by  $4f$  electrons from the nuclear charge

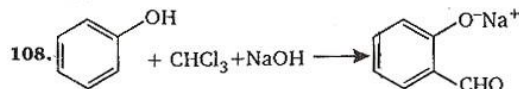
103. Resistance of a conductivity cell filled with a solution of an electrolyte of concentration 0.1 M is  $100 \Omega$ . The conductivity of this solution is  $1.29 \text{ S m}^{-1}$ . Resistance of the same cell when filled with 0.2 M of the same solution is  $520 \Omega$ . The molar conductivity of 0.02 M solution of the electrolyte will be :

- (a)  $124 \times 10^{-4} \text{ S m}^2 \text{ mol}^{-1}$

- (b)  $1240 \times 10^{-4} \text{ S m}^2 \text{ mol}^{-1}$   
 (c)  $1.24 \times 10^{-4} \text{ S m}^2 \text{ mol}^{-1}$   
 (d)  $12.4 \times 10^{-4} \text{ S m}^2 \text{ mol}^{-1}$
104. The ionic mobility of alkali metal ions in aqueous solution is maximum for :  
 (a)  $\text{K}^+$  (b)  $\text{Rb}^+$   
 (c)  $\text{Li}^+$  (d)  $\text{Na}^+$
105. Density of a 2.05 M solution of acetic acid in water is 1.02 g/mL. The molality of the solution is :  
 (a)  $1.14 \text{ mol kg}^{-1}$  (b)  $3.28 \text{ mol kg}^{-1}$   
 (c)  $2.28 \text{ mol kg}^{-1}$  (d)  $0.44 \text{ mol kg}^{-1}$
106. The enthalpy changes for the following processes are listed below :  
 $\text{Cl}_2(\text{g}) = 2\text{Cl}(\text{g}), 242.3 \text{ kJ mol}^{-1}$   
 $\text{I}_2(\text{g}) = 2\text{I}(\text{g}), 151.0 \text{ kJ mol}^{-1}$   
 $\text{ICl}(\text{g}) = \text{I}(\text{g}) + \text{Cl}(\text{g}), 211.3 \text{ kJ mol}^{-1}$   
 $\text{I}_2(\text{s}) = \text{I}_2(\text{g}), 62.76 \text{ kJ mol}^{-1}$
- Given that the standard states for iodine and chlorine are  $\text{I}_2(\text{s})$  and  $\text{Cl}_2(\text{g})$ , the standard enthalpy of formation of  $\text{ICl}(\text{g})$  is :  
 (a)  $-14.6 \text{ kJ mol}^{-1}$  (b)  $-16.8 \text{ kJ mol}^{-1}$   
 (c)  $+16.8 \text{ kJ mol}^{-1}$  (d)  $+244.8 \text{ kJ mol}^{-1}$
107. How many EDTA (ethylenediaminetetra acetic

acid) molecules are required to make an octahedral complex with a  $\text{Ca}^{2+}$  ion ?

- (a) six (b) three  
 (c) one (d) two



The electrophile involved in the above reaction is :

- (a) dichloromethyl cation ( $\text{CHCl}_2^+$ )  
 (b) dichlorocarbene ( $:\text{CCl}_2$ )  
 (c) trichloromethyl anion ( $\text{CCl}_3^-$ )  
 (d) formyl cation ( $\text{CHO}^+$ )

109. 18 g of glucose ( $\text{C}_6\text{H}_{12}\text{O}_6$ ) is added to 178.2 g of water. The vapour pressure of water for this aqueous solution at  $100^\circ\text{C}$  is :  
 (a) 759.00 torr (b) 7.60 torr  
 (c) 76.00 torr (d) 752.40 torr
110. ( $\Delta H - \Delta U$ ) for the formation of carbon monoxide (CO) from its elements at 298 K is : ( $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$ )  
 (a)  $-1238.78 \text{ J mol}^{-1}$  (b)  $1238.78 \text{ J mol}^{-1}$   
 (c)  $-2477.57 \text{ J mol}^{-1}$  (d)  $2477.57 \text{ J mol}^{-1}$

## Part-C

### Mathematics

111. If the roots of the quadratic equation  $x^2 + px + q = 0$  are  $\tan 30^\circ$  and  $\tan 15^\circ$  respectively, then the value of  $2 + q - p$  is :  
 (a) 3 (b) 0  
 (c) 1 (d) 2
112. The value of the integral  $\int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$  is :  
 (a)  $\frac{3}{2}$  (b) 2  
 (c) 1 (d)  $\frac{1}{2}$
113. Let  $W$  denote the words in the English dictionary. Define the relation  $R$  by :  
 $R = \{(x, y) \in W \times W \mid \text{the words } x \text{ and } y \text{ have at least one letter in common}\}$ . Then  $R$  is :  
 (a) reflexive, symmetric and not transitive  
 (b) reflexive, symmetric and transitive  
 (c) reflexive, not symmetric and transitive  
 (d) not reflexive, symmetric and transitive
114. The number of values of  $x$  in the interval  $[0, 3\pi]$  satisfying the equation  $2 \sin^2 x + 5 \sin x - 3 = 0$  is :  
 (a) 6 (b) 1  
 (c) 2 (d) 4
115. If  $A$  and  $B$  are square matrices of size  $n \times n$  such that  $A^2 - B^2 = (A - B)(A + B)$ , then which of the following will be always true ?  
 (a)  $AB = BA$   
 (b) either of  $A$  or  $B$  is a zero matrix  
 (c) either of  $A$  or  $B$  is an identity matrix  
 (d)  $A = B$
116. The value of  $\sum_{k=1}^{10} \left( \sin \frac{2k\pi}{11} + i \cos \frac{2k\pi}{11} \right)$  is :  
 (a) 1 (b)  $-1$   
 (c)  $-i$  (d)  $i$

any three vectors such that  $\vec{a} \cdot \vec{b} \neq 0$ ,  $\vec{b} \cdot \vec{c} \neq 0$ , then  $\vec{a}$  and  $\vec{c}$  are :

- (a) inclined at an angle of  $\frac{\pi}{6}$  between them  
 (b) perpendicular  
 (c) parallel  
 (d) inclined at an angle of  $\frac{\pi}{3}$  between them

118. All the values of  $m$  for which both roots of the equation  $x^2 - 2mx + m^2 - 1 = 0$  are greater than  $-2$  but less than  $4$  lie in the interval :

- (a)  $m > 3$  (b)  $-1 < m < 3$   
 (c)  $1 < m < 4$  (d)  $-2 < m < 0$

119.  $ABC$  is triangle, right angled at  $A$ . The resultant of the forces acting along  $\vec{AB}$ ,  $\vec{AC}$  with magnitudes  $\frac{1}{AB}$  and  $\frac{1}{AC}$  respectively is the force along  $\vec{AD}$ , where  $D$  is the foot of the perpendicular from  $A$  onto  $BC$ . The magnitude of the resultant is :

- (a)  $\frac{(AB)(AC)}{AB + AC}$  (b)  $\frac{1}{AB} + \frac{1}{AC}$   
 (c)  $\frac{1}{AD}$  (d)  $\frac{AB^2 + AC^2}{(AB)^2 (AC)^2}$

120. Suppose a population  $A$  has 100 observations 101, 102, ..., 200 and another population  $B$  has 100 observations 151, 152, ..., 250. If  $V_A$  and  $V_B$  represent the variances of the two population respectively, then  $\frac{V_A}{V_B}$  is :

- (a)  $\frac{9}{4}$  (b)  $\frac{4}{9}$   
 (c)  $\frac{2}{3}$  (d) 1

121.  $\int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2(x + 3\pi)] dx$  is equal to :

- (a)  $\left(\frac{\pi^4}{32}\right) + \left(\frac{\pi}{2}\right)$  (b)  $\frac{\pi}{2}$   
 (c)  $\left(\frac{\pi}{4}\right) - 1$  (d)  $\frac{\pi^4}{32}$

122. In an ellipse, the distances between its foci is 6 and minor axis is 8. Then its eccentricity is :

- (a)  $\frac{1}{2}$  (b)  $\frac{4}{5}$   
 (c)  $\frac{1}{\sqrt{5}}$  (d)  $\frac{3}{5}$

123. The locus of the vertices of the family of

parabolas  $y = \frac{a^3 x^2}{3} + \frac{a^2 x}{2} - 2a$  is :

- (a)  $xy = \frac{3}{4}$  (b)  $xy = \frac{35}{16}$   
 (c)  $xy = \frac{64}{105}$  (d)  $xy = \frac{105}{64}$

124. A straight line through the point  $A(3, 4)$  is such that its intercept between the axes is bisected at  $A$ . Its equation is :

- (a)  $3x - 4y + 7 = 0$  (b)  $4x + 3y = 24$   
 (c)  $3x + 4y = 25$  (d)  $x + y = 7$

125. The value of  $a$ , for which the points  $A, B, C$  with position vectors  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} - 5\hat{k}$  and  $a\hat{i} - 3\hat{j} - \hat{k}$  respectively are the vertices of a right angled triangle with  $C = \frac{\pi}{2}$  are :

- (a)  $-2$  and  $-1$  (b)  $-2$  and  $1$   
 (c)  $2$  and  $-1$  (d)  $2$  and  $1$

126.  $\int_0^{\pi} x f(\sin x) dx$  is equal to :

- (a)  $\pi \int_0^{\pi} f(\sin x) dx$  (b)  $\frac{\pi}{2} \int_0^{\pi/2} f(\sin x) dx$   
 (c)  $\pi \int_0^{\pi/2} f(\cos x) dx$  (d)  $\pi \int_0^{\pi} f(\cos x) dx$

127. The two lines  $x = ay + b$ ,  $z = cy + d$  and  $x = a'y + b'$ ,  $z = c'y + d'$  are perpendicular to each other, if :

- (a)  $aa' + cc' = 1$  (b)  $\frac{a}{a'} + \frac{c}{c'} = -1$   
 (c)  $\frac{a}{a'} + \frac{c}{c'} = 1$  (d)  $aa' + cc' = -1$

128. At an election, a voter may vote for any number of candidates not greater than the number to be elected. There are 10 candidates and 4 are to be elected. If a voter votes for at least one candidate, then the number of ways in which he can vote, is :

- (a) 6210 (b) 385  
 (c) 1110 (d) 5040

129. If the expansion in powers of  $x$  of the function  $\frac{1}{(1-ax)(1-bx)}$  is

- $a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$ , then  $a_n$  is :  
 (a)  $\frac{a^n - b^n}{b - a}$  (b)  $\frac{a^{n+1} - b^{n+1}}{b - a}$   
 (c)  $\frac{b^{n+1} - a^{n+1}}{b - a}$  (d)  $\frac{b^n - a^n}{b - a}$

130. For natural numbers  $m, n$  if



- $(1 - y)^m (1 + y)^n = 1 + a_1 y + a_2 y^2 + \dots$  and  $a_1 = a_2 = 10$ , then  $(m, n)$  is :
- (a) (35, 20)                      (b) (45, 35)  
(c) (35, 45)                      (d) (20, 45)
- 131.** A particle has two velocities of equal magnitude inclined to each other at an angle  $\theta$ . If one of them is halved, the angle between the other and the original resultant velocity is bisected by the new resultant. Then  $\theta$  is :
- (a)  $120^\circ$                       (b)  $45^\circ$   
(c)  $60^\circ$                       (d)  $90^\circ$
- 132.** At a telephone enquiry system the number of phone calls regarding relevant enquiry follow Poisson distribution with an average of 5 phone calls during 10 minute time intervals. The probability that there is at the most one phone call during a 10 minute time period, is :
- (a)  $\frac{5}{6}$                               (b)  $\frac{6}{55}$   
(c)  $\frac{6}{e^5}$                               (d)  $\frac{6}{5^e}$
- 133.** A body falling from rest under gravity passes a certain point  $P$ . It was at a distance of 400 m from  $P$ , 4 seconds prior to passing through  $P$ . If  $g = 10 \text{ m/s}^2$ , then the height above the point  $P$  from where the body began to fall is :
- (a) 900 m                      (b) 320 m  
(c) 680 m                      (d) 720 m
- 134.** The set of points, where  $f(x) = \frac{x}{1 + |x|}$  is differentiable, is :
- (a)  $(-\infty, -1) \cup (-1, \infty)$   
(b)  $(-\infty, \infty)$   
(c)  $(0, \infty)$   
(d)  $(-\infty, 0) \cup (0, \infty)$
- 135.** Let  $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$ ,  $a, b \in N$ . Then :
- (a) there exist more than one but finite number of  $B$ 's such that  $AB = BA$   
(b) there exists exactly one  $B$  such that  $AB = BA$   
(c) there exists infinitely many  $B$ 's such that  $AB = BA$   
(d) there cannot exist any  $B$  such that  $AB = BA$
- 136.** Let  $a_1, a_2, a_3, \dots$  cannot be terms of an AP. If  $a_1 + a_2 + \dots + a_p = \frac{p^2}{q}$ ,  $p \neq q$ , then  $\frac{a_6}{a_{21}}$  equals:
- (a)  $\frac{7}{2}$                               (b)  $\frac{2}{7}$   
(c)  $\frac{11}{41}$                               (d)  $\frac{41}{11}$
- 137.** The function  $f(x) = \frac{x}{2} + \frac{2}{x}$  has a local minimum at :
- (a)  $x = -2$                       (b)  $x = 0$   
(c)  $x = 1$                       (d)  $x = 2$
- 138.** Angle between the tangents to the curve  $y = x^2 - 5x + 6$  at the points (2, 0) and (3, 0) is :
- (a)  $\pi/2$                       (b)  $\pi/6$   
(c)  $\pi/4$                       (d)  $\pi/3$
- 139.** If  $x$  is real, the maximum value of  $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$  is :
- (a) 41                              (b) 1  
(c)  $17/7$                       (d)  $1/4$
- 140.** A triangular park is enclosed on two sides by a fence and on the third side by a straight river bank. The two sides having fence are of same length  $x$ . The maximum area enclosed by the park is :
- (a)  $\sqrt{\frac{x^3}{8}}$                       (b)  $\frac{1}{2} x^2$   
(c)  $\pi x^2$                       (d)  $\frac{3}{2} x^2$
- 141.** If  $(a, a^2)$  falls inside the angle made by the lines  $y = \frac{x}{2}$ ,  $x > 0$  and  $y = 3x$ ,  $x > 0$ , then  $a$  belongs to :
- (a)  $(3, \infty)$                       (b)  $(\frac{1}{2}, 3)$   
(c)  $(-3, -\frac{1}{2})$                       (d)  $(0, \frac{1}{2})$
- 142.** If  $x^m y^n = (x + y)^{m+n}$ , then  $\frac{dy}{dx}$  is :
- (a)  $\frac{x + y}{xy}$                       (b)  $xy$   
(c)  $\frac{x}{y}$                               (d)  $\frac{y}{x}$
- 143.** If the lines  $3x - 4y - 7 = 0$  and  $2x - 3y - 5 = 0$  are two diameters of a circle of area  $49\pi$  square units, the equation of the circle is :
- (a)  $x^2 + y^2 + 2x - 2y - 62 = 0$   
(b)  $x^2 + y^2 - 2x + 2y - 62 = 0$   
(c)  $x^2 + y^2 - 2x + 2y - 47 = 0$   
(d)  $x^2 + y^2 + 2x - 2y - 47 = 0$
- 144.** The image of the point  $(-1, 3, 4)$  in the plane  $x - 2y = 0$  is :
- (a) (15, 11, 4)                      (b)  $(-\frac{17}{3}, -\frac{19}{3}, 1)$   
(c) (8, 4, 4)                      (d)  $(-\frac{17}{3}, -\frac{19}{3}, 4)$

145. The differential equation whose solution is  $Ax^2 + By^2 = 1$ , where  $A$  and  $B$  are arbitrary constant, is of :
- (a) first order and second degree  
 (b) first order and first degree  
 (c) second order and first degree  
 (d) second order and second degree
146. The value of  $\int_1^a [x] f'(x) dx$ ,  $a > 1$ , where  $[x]$  denotes the greatest integer not exceeding  $x$ , is :
- (a)  $[a] f(a) - \{f(1) + f(2) + \dots + f([a])\}$   
 (b)  $[a] f([a]) - \{f(1) + f(2) + \dots + f(a)\}$   
 (c)  $a f([a]) - \{f(1) + f(2) + \dots + f(a)\}$   
 (d)  $a f(a) - \{f(1) + f(2) + \dots + f([a])\}$
147. Let  $C$  be the circle with centre  $(0, 0)$  and radius 3 units. The equation of the locus of the mid points of the chords of the circle  $C$  that subtend an angle of  $\frac{2\pi}{3}$  at its centre, is :
- (a)  $x^2 + y^2 = 1$       (b)  $x^2 + y^2 = \frac{27}{4}$   
 (c)  $x^2 + y^2 = \frac{9}{4}$       (d)  $x^2 + y^2 = \frac{3}{2}$
148. If  $a_1, a_2, \dots, a_n$  are in HP, then the expression  $a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n$  is equal to :
- (a)  $(n-1)(a_1 - a_n)$       (b)  $n a_1 a_n$   
 (c)  $(n-1) a_1 a_n$       (d)  $n(a_1 - a_n)$
149. If  $z^2 + z + 1 = 0$ , where  $z$  is complex number, then the value of  $\left(z + \frac{1}{z}\right)^2 + \left(z^2 + \frac{1}{z^2}\right)^2 + \left(z^3 + \frac{1}{z^3}\right)^2 + \dots + \left(z^6 + \frac{1}{z^6}\right)^2$  is :
- (a) 54      (b) 6  
 (c) 12      (d) 18
150. If  $0 < x < \pi$  and  $\cos x + \sin x = \frac{1}{2}$ , then  $\tan x$  is :
- (a)  $\frac{4 - \sqrt{7}}{3}$   
 (b)  $-\frac{4 + \sqrt{7}}{3}$   
 (c)  $\frac{1 + \sqrt{7}}{4}$   
 (d)  $\frac{1 - \sqrt{7}}{4}$

## ANSWERS

- |          |          |          |          |          |          |          |          |
|----------|----------|----------|----------|----------|----------|----------|----------|
| 1. (d)   | 2. (b)   | 3. (d)   | 4. (b)   | 5. (c)   | 6. (a)   | 7. (b)   | 8. (c)   |
| 9. (d)   | 10. (b)  | 11. (a)  | 12. (a)  | 13. (b)  | 14. (b)  | 15. (d)  | 16. (a)  |
| 17. (c)  | 18. (c)  | 19. (a)  | 20. (d)  | 21. (a)  | 22. (b)  | 23. (b)  | 24. (c)  |
| 25. (b)  | 26. (c)  | 27. (c)  | 28. (b)  | 29. (a)  | 30. (d)  | 31. (b)  | 32. (a)  |
| 33. (c)  | 34. (b)  | 35. (b)  | 36. (a)  | 37. (b)  | 38. (d)  | 39. (*)  | 40. (b)  |
| 41. (c)  | 42. (c)  | 43. (b)  | 44. (c)  | 45. (c)  | 46. (a)  | 47. (d)  | 48. (a)  |
| 49. (a)  | 50. (a)  | 51. (c)  | 52. (a)  | 53. (c)  | 54. (a)  | 55. (c)  | 56. (d)  |
| 57. (c)  | 58. (c)  | 59. (b)  | 60. (d)  | 61. (a)  | 62. (d)  | 63. (c)  | 64. (a)  |
| 65. (d)  | 66. (d)  | 67. (a)  | 68. (b)  | 69. (a)  | 70. (c)  | 71. (d)  | 72. (a)  |
| 73. (c)  | 74. (a)  | 75. (a)  | 76. (b)  | 77. (b)  | 78. (c)  | 79. (d)  | 80. (d)  |
| 81. (a)  | 82. (c)  | 83. (a)  | 84. (d)  | 85. (b)  | 86. (b)  | 87. (d)  | 88. (a)  |
| 89. (c)  | 90. (b)  | 91. (d)  | 92. (d)  | 93. (c)  | 94. (a)  | 95. (d)  | 96. (b)  |
| 97. (a)  | 98. (a)  | 99. (d)  | 100. (d) | 101. (d) | 102. (d) | 103. (d) | 104. (b) |
| 105. (c) | 106. (c) | 107. (c) | 108. (b) | 109. (d) | 110. (b) | 111. (a) | 112. (a) |
| 113. (a) | 114. (d) | 115. (a) | 116. (c) | 117. (c) | 118. (b) | 119. (c) | 120. (d) |
| 121. (b) | 122. (d) | 123. (d) | 124. (b) | 125. (d) | 126. (c) | 127. (d) | 128. (b) |
| 129. (c) | 130. (c) | 131. (a) | 132. (c) | 133. (d) | 134. (b) | 135. (c) | 136. (c) |
| 137. (d) | 138. (a) | 139. (a) | 140. (b) | 141. (b) | 142. (d) | 143. (c) | 144. (*) |
| 145. (c) | 146. (a) | 147. (c) | 148. (c) | 149. (c) | 150. (b) |          |          |

\* No option is matching

# HINTS & SOLUTIONS

## Part-A

### Physics

1. Kirchhoff's 1st law or KCL states that the algebraic sum of current meeting at any junction is equal to zero. In other words we can say that "the sum of all the currents directed towards a junction in a circuit is equal to the sum of all the currents directed away from that junction." Thus, no charge has been accumulated at any junction i.e., charge is conserved, and hence, we can say that KCL ( $\sum i = 0$ ) is based on conservation of charge.

Kirchhoff's 2nd law or KVL states that algebraic sum of changes in potential around any closed resistor loop must be zero. In other words "around any closed loop, voltage drops are equal to voltage rises". No energy is gained or lost in circulating a charge around a loop, thus, we can say that KVL is based on conservation of energy.

2. Ferromagnetic substances have strong tendency to get magnetised (induced magnetic moment) in the same direction as that of applied magnetic field, so magnet attracts  $N_1$  strongly. Paramagnetic substances get weakly magnetised (magnetic moment induced is small) in the same direction as that of applied magnetic field, so magnet attracts  $N_2$  weakly. Diamagnetic substances also get weakly magnetised when placed in an external magnetic field but in opposite direction and hence,  $N_3$  is weakly repelled by magnet.

3. Let  $(\rho_A, l_A, r_A, A_A)$  and  $(\rho_B, l_B, r_B, A_B)$  are specific resistances, lengths, radii and areas of wires A and B respectively.

$$\text{Resistance of A} = R_A = \frac{\rho_A l_A}{A_A} = \frac{\rho_A l_A}{\pi r_A^2}$$

$$\text{Resistance of B} = R_B = \frac{\rho_B l_B}{A_B} = \frac{\rho_B l_B}{\pi r_B^2}$$

From given information

$$\rho_B = 2\rho_A$$

$$r_B = 2r_A$$

and

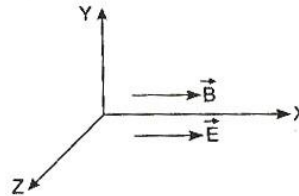
$$R_A = R_B$$

$$\frac{\rho_A l_A}{\pi r_A^2} = \frac{\rho_B l_B}{\pi r_B^2}$$

$$\Rightarrow \frac{\rho_A l_A}{\pi r_A^2} = \frac{2\rho_A \times l_B}{\pi(2r_A)^2}$$

$$\Rightarrow \frac{l_B}{l_A} = \frac{2}{1} = 2:1$$

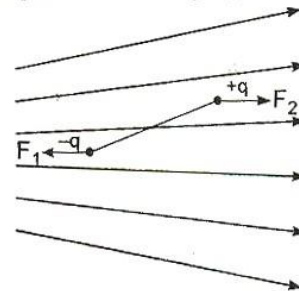
4. Let  $\vec{E}$  and  $\vec{B}$  be along X-axis. When a charged particle is released from rest, it will experience an electric force along the direction of electric field or opposite to the direction of electric field depending on the nature of charge. Due to this force, it acquires some velocity along X-axis. Due to this motion of charge, magnetic force can not have non-zero value because angle between  $\vec{v}$  and  $\vec{B}$  would be either  $0^\circ$  or  $180^\circ$ .



So, only electric force is acting on particle and hence, it will move along a straight line.

5. In a non-uniform electric field, the dipole may experience both non-zero torque as well as translational force.

For example as shown in figure,



$F_1 \neq F_2$  as  $E$  is non-uniform.

Torque would also be non-zero.



6.  $v = \alpha\sqrt{x}$

$$\frac{dx}{dt} = \alpha\sqrt{x} \quad \left(\because v = \frac{dx}{dt}\right)$$

$$\frac{dx}{\sqrt{x}} = \alpha dt$$

Perform integration

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha dt$$

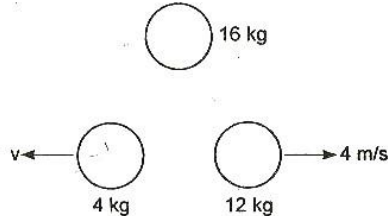
[ $\because$  at  $t = 0$ ,  $x = 0$  and let at any time  $t$ , particle is at  $x$ ]

$$\Rightarrow \frac{x^{1/2}}{1/2} \Big|_0^x = \alpha t$$

$$\Rightarrow x^{1/2} = \frac{\alpha}{2} t$$

$$\Rightarrow x = \frac{\alpha^2}{4} t^2 \Rightarrow x \propto t^2$$

7. Here momentum of the system is remaining conserved as no external force is acting on the bomb (system).



Initial momentum (before explosion) = Final momentum (after explosion)

Let velocity of 4 kg mass is  $v$  m/s. From momentum conservation we can say that its direction is opposite to velocity of 12 kg mass.

$$\text{From } \vec{P}_i = \vec{P}_f$$

$$\Rightarrow 0 = 12 \times 4 - 4 \times v \Rightarrow v = 12 \text{ m/s}$$

$$\text{KE of 4 kg mass} = \frac{4 \times (12)^2}{2} = 288 \text{ J}$$

8. Velocity of sound in air = 300 m/s

If a source of sound is moving towards a stationary listener, the frequency heard by the listener would be different from the actual frequency of the source, this apparent frequency is given by

$$f_{\text{app.}} = \left( \frac{v_{\text{sound in air}}}{v_{\text{sound in air}} \pm v_{\text{source}}} \right), \text{ where symbols}$$

have their usual meanings.

In the denominator +ve sign would be taken when source is receding away from the listener, while -ve sign would be taken when source is approaching the listener.

Let  $v$  be the maximum value of source velocity for which the person is able to hear the sound, then

$$10,000 = f_{\text{app.}} = \left( \frac{300}{300 - v} \right) \times 9500$$

$$\Rightarrow v = 15 \text{ m/s}$$

9. Here, the constant horizontal force required to take the body from position 1 to position 2 can be calculated by using work-energy theorem. Let us assume that body is taken slowly so that its speed doesn't change, then  $\Delta K = 0$

$$= W_F + W_{Mg} + W_{\text{tension}}$$

[symbols have their usual meanings]

$$W_F = F \times l \sin 45^\circ,$$

$$W_{Mg} = Mg(l - l \cos 45^\circ), W_{\text{tension}} = 0$$

$$\therefore F = Mg(\sqrt{2} - 1)$$

10. The height ( $h$ ) traversed by particle while going up is

$$h = \frac{u^2}{2g} = \frac{25}{2 \times 9.8} \quad \text{O } v = 0$$

$$\text{Work done by gravity force} = m \vec{g} \cdot \vec{h}$$

$$= 0.1 \times g \times \frac{25}{2 \times 9.8} \cos 180^\circ \quad \begin{matrix} \uparrow 5 \text{ m/s} \\ \text{O } 100 \text{ g} \end{matrix}$$

[Angle between force and displacement is  $180^\circ$ ]

$$\therefore W = -0.1 \times \frac{25}{2} = -1.25 \text{ J}$$

11. The maximum velocity of a particle performing SHM is given by  $v = A\omega$ , where  $A$  is the amplitude and  $\omega$  is the angular frequency of oscillation.

$$\therefore 4.4 = (7 \times 10^{-3}) \times 2\pi / T$$

$$\Rightarrow T = \frac{7 \times 10^{-3} \times 2 \times 22}{4.4 \times 7} = 0.01 \text{ s}$$

12. KE of a body undergoing SHM is given by

$$\text{KE} = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t \text{ and } \text{KE}_{\text{max}} = \frac{m\omega^2 A^2}{2}$$

[symbols represent standard quantities]

From given information

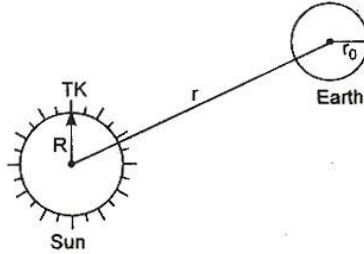
$$\text{KE} = (\text{KE}_{\text{max}}) \times \frac{75}{100}$$

$$\Rightarrow \frac{m\omega^2 A^2}{2} \cos^2 \omega t = \frac{m\omega^2 A^2}{2} \times \frac{3}{4}$$

$$\Rightarrow \cos \omega t = \pm \frac{\sqrt{3}}{2}$$

$$\Rightarrow \omega t = \frac{\pi}{6} \Rightarrow \frac{2\pi}{T} \times t = \frac{\pi}{6} \Rightarrow t = \frac{T}{12} = \frac{1}{6} \text{ s}$$

13. From Stefan's law, the rate at which energy is radiated by sun at its surface is



$$P = \sigma \times 4\pi R^2 \times T^4$$

[Sun is a perfectly black body as it emits radiations of all wavelengths and so for it  $e = 1$ .]  
The intensity of this power at earth's surface [under the assumption  $r \gg r_0$ ] is

$$I = \frac{P}{4\pi r^2} = \frac{\sigma \times 4\pi R^2 T^4}{4\pi r^2} = \frac{\sigma R^2 T^4}{r^2}$$

The area of earth which receives this energy is only one half of total surface area of earth, whose projection would be  $\pi r_0^2$ .

$\therefore$  Total radiant power as received by earth

$$\begin{aligned} &= \pi r_0^2 \times I \\ &= \frac{\pi r_0^2 \times \sigma R^2 T^4}{r^2} \\ &= \frac{\pi r_0^2 R^2 \sigma T^4}{r^2} \end{aligned}$$

14. Magnetic energy =  $\frac{1}{2} Li^2 = \frac{LQ^2}{2t^2}$

[ $L \rightarrow$  inductance,  $i \rightarrow$  current]

Energy has the dimensions  $[ML^2T^{-2}]$ .

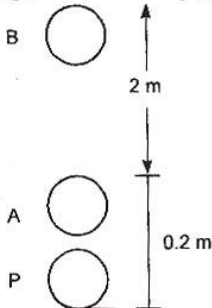
Equate the dimensions, we have

$$[ML^2T^{-2}] = (\text{henry}) \times \frac{[Q^2]}{[T^2]}$$

$$\Rightarrow [\text{henry}] = \frac{[ML^2]}{[Q^2]}$$

15. The situation is shown in figure. At initial time,

the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B let acceleration of ball during PA is  $a \text{ m/s}^2$  [assumed to be constant] in upward direction and velocity of ball at A is  $v \text{ m/s}$ .



Then for PA,  $v^2 = 0^2 + 2a \times 0.2$

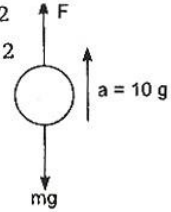
For AB,  $0 = v^2 - 2 \times g \times 2$

$$\Rightarrow v^2 = 2g \times 2$$

From above equations,

$$a = 10g = 100 \text{ m/s}^2$$

Then for PA, FBD of ball is



$F - mg = ma$  [  $F$  is the force exerted by hand on ball ]

$$\Rightarrow F = m(g + a) = 0.2(11g) = 22N$$

**Alternate solution :**

Using work-energy theorem

$$W_{mg} + W_F = 0$$

$$\Rightarrow -mg \times 2.2 + F \times 0.2 = 0 \Rightarrow F = 22N$$

16. For string fixed at both the ends, resonant frequency are given by  $f = \frac{nv}{2L}$ , where symbols

have their usual meanings. It is given that 315 Hz and 420 Hz are two consecutive resonant frequencies, let these are  $n$ th and  $(n + 1)$ th harmonics.

$$315 = \frac{nv}{2L} \quad \dots (i)$$

$$420 = \frac{(n+1)v}{2L} \quad \dots (ii)$$

$$\Rightarrow \text{Eq. (i)} \div \text{Eq. (ii)} \Rightarrow \frac{315}{420} = \frac{n}{n+1} \Rightarrow n = 3$$

From Eq. (i), lowest resonant frequency

$$f_0 = \frac{v}{2L} = \frac{315}{3} = 105 \text{ Hz}$$

17. To keep the COM at the same position, velocity of COM is zero, so

$$\frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2} = 0$$

[where  $\vec{v}_1$  and  $\vec{v}_2$  are velocities of particles 1 and 2 respectively.]

$$\Rightarrow m_1 \frac{d\vec{r}_1}{dt} + m_2 \frac{d\vec{r}_2}{dt} = 0$$

$$[\because \vec{v}_1 = \frac{d\vec{r}_1}{dt} \text{ \& } \vec{v}_2 = \frac{d\vec{r}_2}{dt}]$$

$$\Rightarrow m_1 d\vec{r}_1 + m_2 d\vec{r}_2 = 0 \text{ [} d\vec{r}_1 \text{ and } d\vec{r}_2 \text{ represent the change in displacement of particles]}$$

Let 2nd particle has been displaced by distance  $x$ .

$$\Rightarrow m_1(d) + m_2(x) = 0 \Rightarrow x = -\frac{m_1 d}{m_2}$$

-ve sign shows that both the particles have to move in opposite directions.

So,  $\frac{m_1 d}{m_2}$  is the distance moved by 2nd particle to keep COM at the same position.

18. This is the question based on impulse-momentum theorem.

$$|F \cdot \Delta t| = |\text{change in momentum}|$$

$$\Rightarrow F \times 0.1 = |p_f - p_i|$$

As the ball will stop after catching;

$$p_i = mv_i = 0.15 \times 20 = 3, p_f = 0$$

$$\Rightarrow F \times 0.1 = 3 \Rightarrow F = 30 \text{ N}$$

19.  $\beta = \frac{I_C}{I_B}$  and  $I_E = I_C + I_B$

$$\therefore \beta = \frac{I_C}{I_E - I_C} = \frac{5.488}{5.60 - 5.488} = 49$$

20. When a circuit is made up of any two metals in thermoelectric series, the current flows across the cold junction from the later occurring metal in the series to the one occurring earlier. In thermoelectric series Bismuth comes earlier than Antimony. So, at cold junction current flows from Antimony to Bismuth and at hot junction it flows from Bismuth to Antimony.

21.  $h\nu_0 = 6.2 \text{ eV}$ ,  $eV_0 = 5 \text{ eV}$

From Einstein's photoelectric equation

$$h\nu = h\nu_0 + eV_0 = 6.2 + 5 = 11.2 \text{ eV}$$

$$\Rightarrow \frac{hc}{\lambda} = 11.2 \text{ eV}$$

$$\therefore \lambda = \frac{hc}{11.2(\text{eV})} = 1108.9 \text{ \AA}$$

which belongs to ultra-violet region.

22. Since, here nuclear target is heavy it can be assumed safely that it will remain stationary and will not move due to the Coulombic interaction force.

At distance of closest approach relative velocity of two particles is  $v$ . Here target is considered as stationary, so  $\alpha$ -particle comes to rest instantaneously at distance of closest approach. Let required distance is  $r$ , then from work-energy theorem

$$0 - \frac{mv^2}{2} = -\frac{1}{4\pi\epsilon_0} \frac{Ze \times 2e}{r}$$

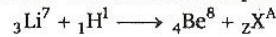
$$\Rightarrow r \propto \frac{1}{m}$$

$$\propto \frac{1}{v^2}$$

$$\propto Ze^2$$

23. The photoelectric effect is an instantaneous phenomenon (experimentally proved). It takes approx. time of the order of  $10^{-10}$  s.

24. The nuclear reaction can be represented as



Applying conservation of atomic number (charge)

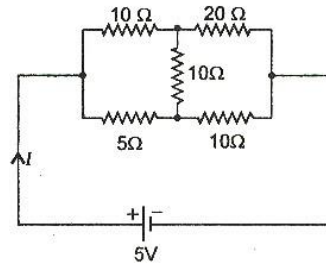
$$3 + 1 = 4 + Z \Rightarrow Z = 0$$

Applying conservation of atomic mass

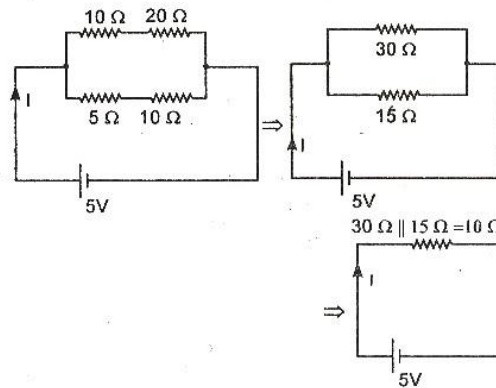
$$7 + 1 = 8 + A \Rightarrow A = 0$$

Thus, the emitted particles are  $\gamma$ -photons ( ${}_0\text{X}^0$ ).

25. The given circuit can be redrawn as

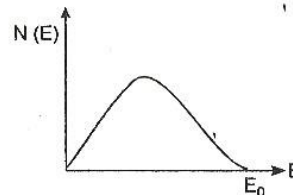


which is a balanced Wheatstone's bridge and hence, no current flows in the centre resistor, so equivalent circuit would be as shown below.



$$\text{So, } I = \frac{V}{R} = \frac{5}{10} = 0.5 \text{ A}$$

26. Energy spectrum of emitted  $\beta$ -particles from a radioactive source is drawn as





27. At resonance,  $\omega L = \frac{1}{\omega C}$

Current flowing through the circuit,

$$I = \frac{V_R}{R} = \frac{100}{1000} = 0.1 \text{ A}$$

So, voltage across  $L$  is given by

$$V_L = I X_L = I \omega L$$

but  $\omega L = \frac{1}{\omega C}$

$$\therefore V_L = \frac{I}{\omega C} = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250 \text{ V}$$

28. Let resistance of bulb filament is  $R_0$  at  $0^\circ\text{C}$  then from expression

$$R_\theta = R_0[1 + \alpha\Delta\theta]$$

we have,  $100 = R_0[1 + 0.005 \times 100]$

and  $200 = R_0[1 + 0.005 \times x]$

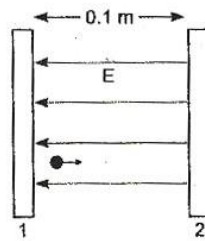
where  $x$  is temperature in  $^\circ\text{C}$  at which resistance become  $200 \Omega$ .

Dividing the above two equations

$$\frac{200}{100} = \frac{1 + 0.005x}{1 + 0.005 \times 100} \Rightarrow x = 400^\circ\text{C}$$

29. Since  $V_2 > V_1$ , so electric field will point from plate 2 to plate 1.

The electron will experience an electric force, opposite to the direction of electric field, and hence move towards the plate 2.



Use work-energy theorem to find speed of electron when it strikes the plate 2.

$$\frac{m_e v^2}{2} - 0 = e(V_2 - V_1)$$

where  $v$  is the required speed.

$$\therefore \frac{9.11 \times 10^{-31}}{2} v^2 = 1.6 \times 10^{-19} \times 20$$

$$\Rightarrow v = \sqrt{\frac{1.6 \times 10^{-19} \times 40}{9.11 \times 10^{-31}}} = 2.65 \times 10^6 \text{ m/s}$$

30. The emf generated would be maximum when flux (cutting) would be maximum i.e., angle between area vector of coil and magnetic field is  $0^\circ$ . The emf generated is given by [as a function of time]

$$e = NBA\omega \cos \omega t$$

$$\Rightarrow e_{\text{max}} = NAB\omega$$

32.  $D = (\mu - 1)A$

For blue light  $\mu$  is greater than that for red light, so  $D_2 > D_1$ .

33.  $I = n_e A v_d$

$$\therefore \frac{I_e}{I_h} = \frac{n_e \times (v_d)_e}{n_h \times (v_d)_h}$$

Here,  $\frac{n_e}{n_h} = \frac{7}{5}$ ,  $\frac{I_e}{I_h} = \frac{7}{4}$

$$\therefore \frac{7}{4} = \frac{7}{5} \times \frac{(v_d)_e}{(v_d)_h}$$

$$\Rightarrow \frac{(v_d)_e}{(v_d)_h} = \frac{5}{7} \times \frac{7}{4} = \frac{5}{4}$$

34. For balanced Wheatstone's bridge

$$\frac{P}{Q} = \frac{R}{S} \text{ here } S = S_1 \parallel S_2 = \frac{S_1 S_2}{S_1 + S_2}$$

$$\Rightarrow \frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$$

35.  $\phi = 10t^2 - 50t + 250$

From Faraday's law of electromagnetic induction  $e = -\frac{d\phi}{dt}$

$$\therefore e = -[10 \times 2t - 50]$$

$$\therefore e|_{t=3s} = -[10 \times 6 - 50] = -10 \text{ V}$$

36. Magnetic field due to a long solenoid is given by

$$B = \mu_0 n i$$

From given data,

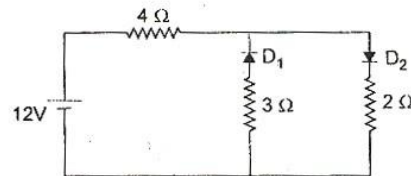
$$6.28 \times 10^{-2} = \mu_0 \times 200 \times 10^2 \times i \dots (i)$$

and  $B = \mu_0 \times 100 \times 10^2 \times (i/3) \dots (ii)$

Solving Eqs.(i) and (ii), we get

$$B \approx 1.05 \times 10^{-2} \text{ Wb/m}^2$$

37. In the given circuit diode  $D_1$  is reverse biased while  $D_2$  is forward biased, so the circuit can be redrawn as :



[ $\therefore$  For ideal diodes, reverse biased means open and forward biased means short]

Apply KVL to get current flowing through the circuit

$$-12 + 4i + 2i = 0 \Rightarrow i = \frac{12}{6} = 2 \text{ A}$$

38. For reverse biasing of an ideal diode, the potential of  $n$ -side should be higher than potential of  $p$ -side. Only option (d) is satisfying the criterion for reverse biasing.

39. According to the photoelectric effect in a photocell, if a light of wavelength  $\lambda$  is incident on a cathode, then electrons are emitted, which constitute the photoelectric current.

Photocell is based on the principle of photoelectric effect. As the wavelength of light changes, there is no change in number of electrons emitted and hence, no change in current (plate current of photocell). Thus, the two wavelength of incident light and plate current are independent to each other.

Plate current depends on intensity of light used.

**Note :** Here, no option is matching.

40. Energy of proton  $= 7 \times 5.60 = 2 \times [4 \times 7.06]$   
 $\therefore$  Energy of proton  $= 17.28 \text{ MeV}$

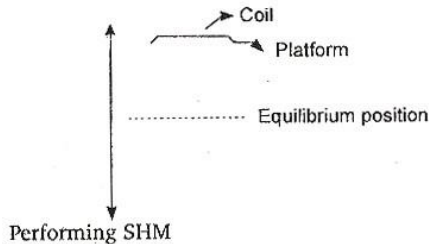
41. Resistance of electric bulb  $R = \frac{V^2}{P}$  where subscripts denote for rated parameters.

$$R = \frac{(220)^2}{100}$$

Power consumed at 110 volt,  $P_{\text{consumed}} = \frac{V^2}{R}$

$$\therefore P_{\text{consumed}} = \frac{(110)^2}{(220)^2 / 100} = 25 \text{ W}$$

42. 'rad' is used to measure biological effect of radiation.
43. As the amplitude is increased, the max. acceleration of the platform (along with coin as long as they doesn't get separated) increases.



If we draw the FBD for coin at one of the extreme positions as shown



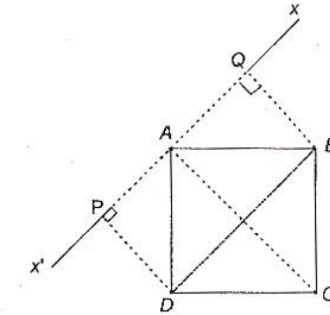
then from Newton's law,  $mg - N = m\omega^2 A$

For losing contact with the platform,  $N = 0$

So,  $A = g/\omega^2$

44. The situation is shown in figure.

$$I_{xx'} = m \times DP^2 + m \times BQ^2 + m \times CA^2$$



$$= m \times 2 \times \left(\frac{\sqrt{2}l}{2}\right)^2 + m \times (\sqrt{2}l)^2$$

$$= 3ml^2$$

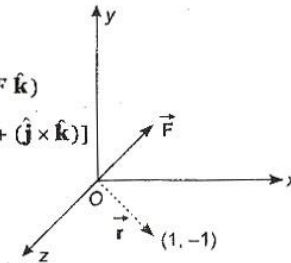
45.  $\vec{\tau} = \vec{r} \times \vec{F}$

$$\vec{\tau} = (\hat{i} - \hat{j}) \times (-F \hat{k})$$

$$= F[(-\hat{i} \times \hat{k}) + (\hat{j} \times \hat{k})]$$

$$= F[\hat{j} + \hat{i}]$$

$$= F[\hat{i} + \hat{j}]$$



46.  $V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$

For minimum value of  $V$ ,  $\frac{dV}{dx} = 0$

$$\Rightarrow \frac{4x^3}{4} - \frac{2x}{2} = 0 \Rightarrow x = 0, x = \pm 1$$

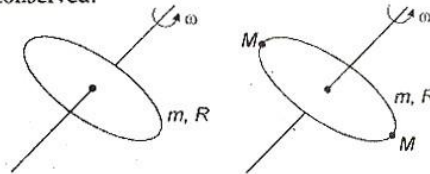
So,  $V_{\text{min.}} (x = \pm 1) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4} \text{ J}$

$\therefore K_{\text{max.}} + V_{\text{min.}} = \text{Total mechanical energy}$

$$K_{\text{max.}} = (1/4) + 2 \Rightarrow K_{\text{max.}} = 9/4$$

$$\Rightarrow \frac{mv^2}{2} = \frac{9}{4} \Rightarrow v = \frac{3}{\sqrt{2}} \text{ m/s}$$

47. As no external torque is acting about the axis, angular momentum of system remains conserved.

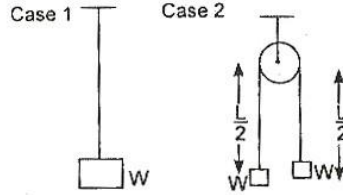


$$\therefore I_1 \omega = I_2 \omega'$$

$$\Rightarrow mR^2 \omega = (mR^2 + 2MR^2) \omega'$$

$$\Rightarrow \omega' = \left(\frac{m}{m + 2M}\right) \omega$$

48. Let us consider the length of wire as  $L$  and cross-sectional area  $A$ , the material of wire has Young's modulus as  $Y$ .



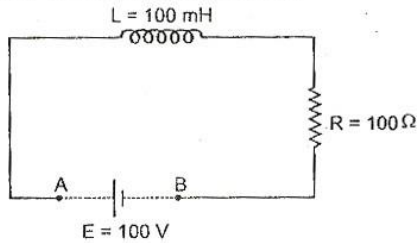
Then for 1st case  $Y = \frac{W/A}{l/L}$

For 2nd case,  $Y = \frac{W/A}{2l/L}$

$\therefore l = L/2$

So, total elongation of both sides =  $2l = l$

49. This is a combined example of growth and decay of current in an LR circuit.



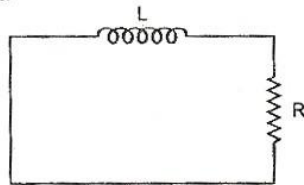
The current through circuit just before shorting the battery,

$$I_0 = E/R = 1A$$

[as inductor would be shorted in steady state]

After this decay of current starts in the circuit according to the equation  $I = I_0 e^{-t/\tau}$  where

$$\tau = L/R.$$



$$I = 1 \times e^{-(1 \times 10^{-3})/(100 \times 10^{-3}/100)}$$

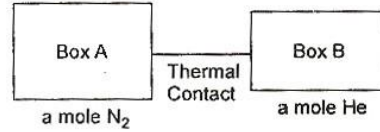
$$= (1/e)A$$

50. When the two conducting spheres are connected by a conducting wire, charge will flow from one sphere (having higher potential) to other (having lower potential) till both acquire the same potential.

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

So,  $\frac{E_1}{E_2} = \left(\frac{r_2}{r_1}\right)^2 = 4:1$

51. Here, change in internal energy of the system is zero. i.e., increase in internal energy of one is equal to decrease in internal energy of other.



$$\Delta U_A = 1 \times \frac{5R}{2} (T_f - T_0)$$

$$\Delta U_B = 1 \times \frac{3R}{2} \left(T_f - \frac{7}{3}T_0\right)$$

Now  $\Delta U_A + \Delta U_B = 0$

$$\frac{5R}{2} (T_f - T_0) + \frac{3R}{2} \left(T_f - \frac{7}{3}T_0\right) = 0$$

$$5T_f - 5T_0 + 3T_f - 7T_0 = 0$$

$$\Rightarrow 8T_f = 12T_0 \Rightarrow T_f = \frac{12}{8}T_0 = \frac{3}{2}T_0$$

52. For adiabatic process,  $dQ = 0$

So,  $dU = -\Delta W$

$$\Rightarrow nC_v dT = +146 \times 10^3 J$$

$$\Rightarrow \frac{nfR}{2} \times 7 = 146 \times 10^3$$

[ $f \rightarrow$  Degree of freedom]

$$\Rightarrow \frac{10^3 \times f \times 8.3 \times 7}{2} = 146 \times 10^3$$

$$f = 5.02 \approx 5.$$

So, it is a diatomic gas.

53. If lattice constant of semiconductor is decreased, then  $E_c$  and  $E_v$  decrease but  $E_g$  increases.

54. Total average energy =  $\epsilon_0 E_{rms}^2$

$$= 8.85 \times 10^{-12} \times (720)^2$$

$$= 4.58 \times 10^{-6} J/m^3$$

55. Terminal speed of spherical body in a viscous liquid is given by

$$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

where  $\rho =$  density of substance of body

$\sigma =$  density of liquid

From given data

$$\frac{v_T(Ag)}{v_T(Gold)} = \frac{\rho_{Ag} - \sigma_l}{\rho_{Gold} - \sigma_l}$$

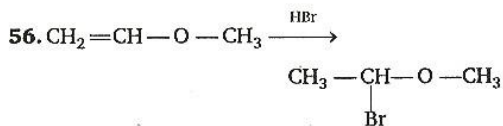
$$\Rightarrow v_T(Ag) = \frac{10.5 - 1.5}{19.5 - 1.5} \times 0.2 = \frac{9}{18} \times 0.2$$

$$= 0.1 m/s$$

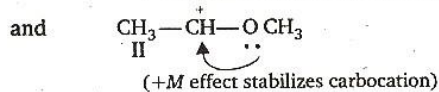
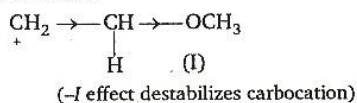


## Part-B

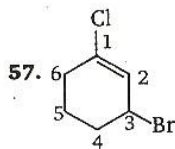
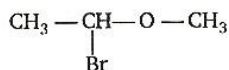
### Chemistry



First protonation occurs, two possible intermediates are



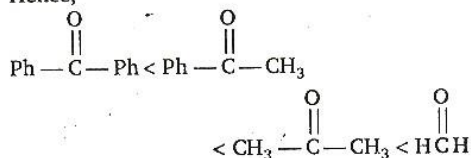
II, is more favourable Hence,  $\text{Br}^\ominus$  attacks, and product is



Unsaturation (double bond) is given priority over halogen, then lowest set of locants. So the correct IUPAC name is 3-bromo-1-chlorocyclohexene

58. Addition of HCN is nucleophilic addition. Greater the electron deficiency of carbonyl group higher the rate of reaction.

Hence,



59.  $\text{Mg}_3(\text{PO}_4)_2$ ; mole

8 moles of O-atom are contained by 1 mole  $\text{Mg}_3(\text{PO}_4)_2$ .

Hence, 0.25 moles of O-atom =  $\frac{1}{8} \times 0.25$   
 $= 3.125 \times 10^{-2}$

60. Angular momentum of an electron

$$= mvr = \frac{nh}{2\pi} \quad (n \text{ is orbit number})$$

$$\text{in 5th orbit} = \frac{5h}{2\pi}$$

$$= \frac{2.5 h}{\pi}$$

61.  $\text{O}_2^{2-}$  (Total number of electrons = 18)

$$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2$$

$$= \pi 2p_y^2, \pi^* 2p_x^2 = \pi^* 2p_y^2$$

62. Total number of atoms in face centred cubic unit cell = 4

At corner =  $8 \times \frac{1}{8} = 1$

On faces =  $6 \times \frac{1}{2} = 3$

Total = 4

Total volume =  $4 \times \frac{4\pi r^3}{3}$   
 $= \frac{16}{3} \pi r^3$

63.

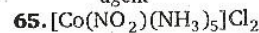
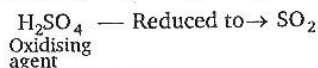
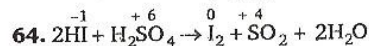
$$r \propto [\text{CO}]^2$$

$$r' \propto [2\text{CO}]^2$$

$$r' \propto 4 [\text{CO}]^2$$

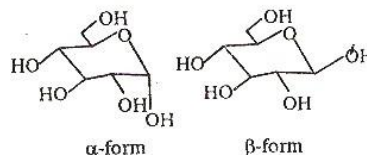
Hence  $\frac{r'}{r} = \frac{4 [\text{CO}]^2}{[\text{CO}]^2}$

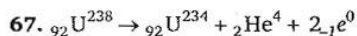
$$r' = 4r$$



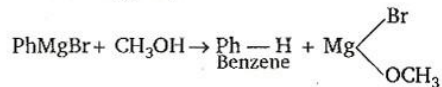
Pentaamine nitrito-N-cobalt (III)chloride

66. Anomers of glucose are cyclic diastereomers (epimers) differing in configuration at C-1, existing in two forms  $\alpha$  and  $\beta$  respectively.

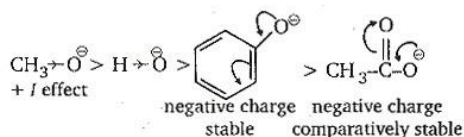




68. PhMgBr can be protonated by any of the protic solvent e.g.,  $\text{CH}_3\text{OH}$ .

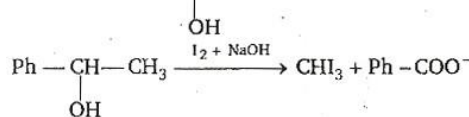


69. Nucleophilicity order is :

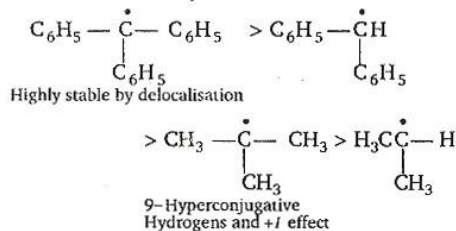


70. In DNA, cytosine and thymine are pyrimidine bases.

71. For positive iodoform test, alcohol molecule must have  $\text{CH}_3\text{-CH(OH)-}$  group

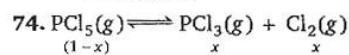


72. Free radicals stability :



73.  $\Delta x \cdot \Delta v \geq \frac{h}{4\pi m}$

$$\Delta x = \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31} \times 300 \times 0.001 \times 10^{-2}} = 0.01933 = 1.93 \times 10^{-2}$$

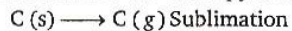


Total number of moles at equilibrium

$$= (1-x) + x + x = 1+x$$

$$P_{\text{PCl}_3} = \left[ \frac{x}{1+x} \right] \times P$$

75. Carbon is found in solid state. The state of substance affects the enthalpy change.



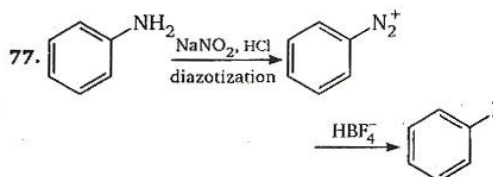
and  $\text{H}_2(\text{g}) \rightarrow 2\text{H}(\text{g})$  dissociation

are required for C—H bond

76. Molecules

Interaction

Benzene and ethanol	dispersion force
Acetonitrile and acetone	dipole-dipole
KCl and water	ion-dipole
Benzene and carbon tetrachloride	dispersion (London) force



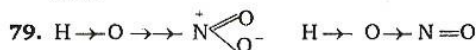
78.  $\text{MCl}_2$  Oxidation state of  $M = +2$

$\text{MCl}_4$  Oxidation state of  $M = +4$

Higher the oxidation state, smaller the size.

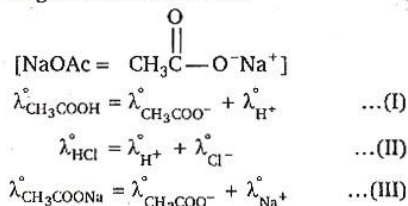
Greater the polarizing power, greater the covalent characteristics

Hence  $\text{MCl}_4$  is more covalent and  $\text{MCl}_2$  is more ionic.



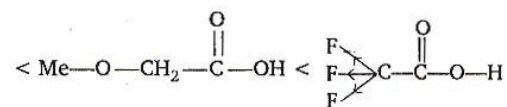
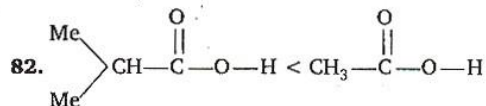
Polarity along O—H in  $\text{HNO}_3$  is more in comparison to —O—H in  $\text{HNO}_2$ .

80. According to Kohlrausch's law-



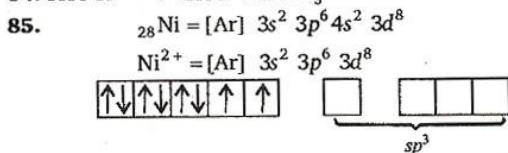
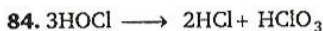
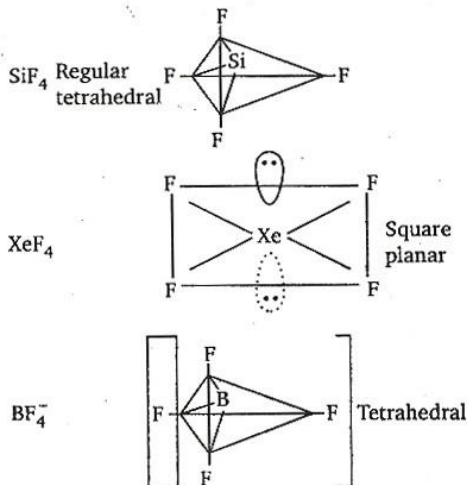
thus on adding (II) and (III) if  $\lambda_{\text{Na}^+}^\circ$  and  $\lambda_{\text{Cl}^-}^\circ$  are subtracted we can obtain the value of  $\lambda_{\text{HOAc}}^\circ$ . Thus, additional value required is  $\lambda_{\text{NaCl}}^\circ$ .

81. isoelectronic means having same no. of electrons.  $\text{K}^+$ ,  $\text{Cl}^-$ ,  $\text{Ca}^{2+}$ ,  $\text{Sc}^{3+}$  (all are having 18 electrons).



- I effect increases acidity.
- +I effect decreases acidity.
- CF<sub>3</sub> exerting more -I effect than MeO-
- Me<sub>2</sub>CH- exerting more +I effect than -CH<sub>3</sub>

83. SF<sub>4</sub> has trigonal bipyramidal geometry, lone pair of electrons repels the axial bond pair and decreases the bond angle to 173°.



Nickel has two unpaired electrons and geometry is tetrahedral due to  $sp^3$  hybridisation.

86. Metal carbonyl organometallic compounds possess both  $\sigma$  and  $\pi$  characters.
87. Examine the positions in periodic table.

B C N O F  
P S

phosphorus is having stable half filled configuration.

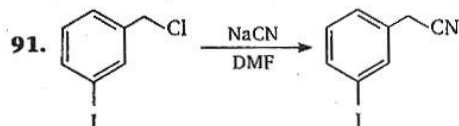
Hence order is  $B < S < P < F$ .

88.  $T_{f(\text{irreversible})} > T_{f(\text{reversible})}$   
 It is an adiabatic expansion and  $W$  (rev) is maximum.

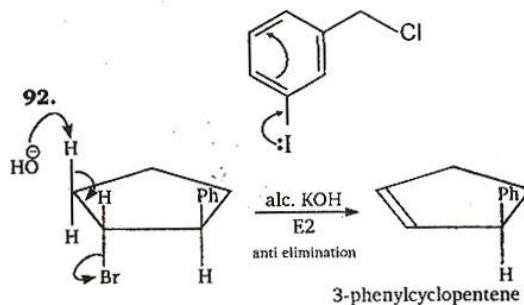
89. The adsorption of a gas is directly proportional to the pressure of the gas.

90.  $k = A e^{-E/RT}$

$E$  is activation energy, it is that energy, which molecule must have to give the product.

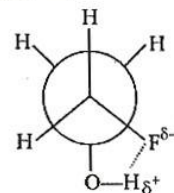


Chloride is on 1° aliphatic carbon which is substituted easier in comparison to iodide which is aryl and more stable due to delocalisation hence difficult to substitute.



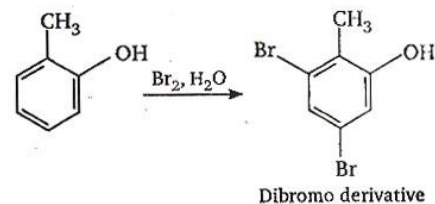
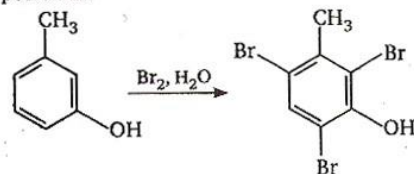
Anti-elimination, means -H and the -Br both departing group must be present at dihedral angle of 180° (anti).

93.  $\text{HO}-\text{CH}_2-\text{CH}_2-\text{F}$



Gauche conformation is comparatively more stable due to hydrogen linkage in between F and H (at O-atom), hence order is Eclipse, Anti (staggered), Gauche.

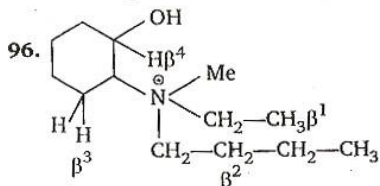
94. *m*-cresol due to phenoxide ion in H<sub>2</sub>O solvent, gives tribromoderivative at all *ortho* and *para* positions.





95.  $\text{NH}_3 > \text{PH}_3 > \text{AsH}_3 > \text{SbH}_3$

As the electronegativity of central atom increases, bonded electron polarise towards central atom more, so, repulsion increases and bond angle increases.

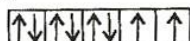


There are four  $\beta$ -hydrogens, in this quaternary ammonium salt.

On heating quaternary ammonium salt gives Hofmann elimination (abstraction of most acidic hydrogen which is  $\beta^1$ )

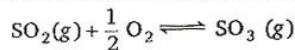
Hence major product is  $\text{CH}_2=\text{CH}_2$ . (Least substituted alkene)

97.  $\text{Ni}^{2+} = [\text{Ar}] 3d^8$



Number of unpaired electrons = 2  
Hence, magnetic moment =  $\sqrt{n(n+2)}$   
 $= \sqrt{8} = 2.84$

98. Equilibrium constant for the reaction :



$$K_c = \frac{1}{4.9 \times 10^{-2}}$$

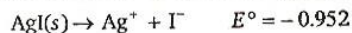
and for  $2\text{SO}_2 + \text{O}_2 \rightleftharpoons 2\text{SO}_3(\text{g})$

$$K_c = \left( \frac{1}{4.9 \times 10^{-2}} \right)^2$$

$$= \frac{10^4}{(4.9)^2} = 416.49$$

99. Alkali metals reactivity increases down the group as electropositivity increases, but for halogens  $\text{F}_2$  is more reactive as moving down molecular stability increases.

100.  $\text{AgI}(\text{s}) + e^- \rightleftharpoons \text{Ag}(\text{s}) + \text{I}^-$ ;  $E^\circ = -0.152$



$$E^\circ_{\text{cell}} = \frac{0.059}{n} \log K_{sp}$$

$$-0.952 = \frac{0.059}{1} \log K_{sp}$$

$$\log K_{sp} = \frac{-0.952}{0.059} = -16.135$$

101. Rate =  $k [\text{NOBr}_2][\text{NO}]$  ... (1)

But  $\text{NOBr}_2$  is in equilibrium.

$$K_{\text{eq}} = \frac{[\text{NOBr}_2]}{[\text{NO}] [\text{Br}_2]}$$

$$[\text{NOBr}_2] = K_{\text{eq}} [\text{NO}] [\text{Br}_2] \quad \dots (2)$$

Putting the  $[\text{NOBr}_2]$  in (1)

$$\text{Rate} = k \cdot K_{\text{eq}} [\text{NO}] [\text{Br}_2] [\text{NO}]$$

$$\text{Hence Rate} = k \cdot K_{\text{eq}} [\text{NO}]^2 [\text{Br}_2]$$

$$\text{Rate} = k' [\text{NO}]^2 [\text{Br}_2]$$

where  $k' = k \cdot K_{\text{eq}}$ .

102. Lanthanoid contraction is due to ineffective shielding produced by larger  $f$ -subshell.

103.  $R = 100 \Omega$

$$\kappa = \frac{1}{R} \left( \frac{l}{a} \right)$$

$$\frac{l}{a} (\text{cell constant}) = 1.29 \times 100 \text{ m}^{-1}$$

Given  $R = 520 \Omega$ ;  $C = 0.2 \text{ M}$

$\mu$  (molar conductivity) = ?

$$\mu = k \times V$$

( $\kappa$  can be calculated as  $\kappa = \frac{1}{R} \left( \frac{l}{a} \right)$  now cell

constant is known)

$$\text{Hence } \mu = \frac{1}{520} \times 129 \times \frac{1000}{0.2} \times 10^{-6} \text{ m}^3$$

$$= 12.4 \times 10^{-4} \text{ Sm}^2 \text{ mol}^{-1}$$

104.  $\text{Li}^+$  is having largest hydrated ionic size while  $\text{Rb}^+$  is having smallest.

Smaller the size, greater the mobility.

105. molality ( $m$ ) =  $\frac{M}{1000d - MM_1} \times 100$

$M$  = Molarity

$M_1$  = Molecular mass

$d$  = density

$$= \frac{2.05}{(1000 \times 1.02) - (2.05 \times 60)} \times 1000$$

$$= 2.28 \text{ mol kg}^{-1}$$

106.  $\frac{1}{2} \text{I}_2(\text{s}) + \frac{1}{2} \text{Cl}_2(\text{g}) \rightarrow \text{ICl}(\text{g})$

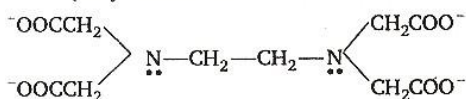
$$\Delta H = \left[ \frac{1}{2} \Delta H_{s \rightarrow g} + \frac{1}{2} \Delta H_{\text{diss.}} (\text{Cl}_2) + \frac{1}{2} \Delta H_{\text{diss.}} (\text{I}_2) \right] - \Delta H_{\text{ICl}}$$

$$= \left( \frac{1}{2} \times 62.76 + \frac{1}{2} \times 242.3 + \frac{1}{2} \times 151.0 \right) - 211.3$$

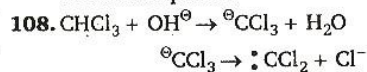
$$= 228.03 - 211.3$$

$$\Delta H = 16.73$$

107. EDTA (Ethylenediaminetetra acetic acid)



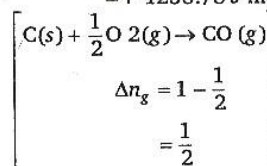
It is hexadentate (6 electron pairs) that's why for octahedral complex only one EDTA is required.



109.  $\frac{P^\circ - P_s}{P_s} = \frac{n}{N}$   
 $\frac{760 - P_s}{P_s} = \frac{18/180}{178.2/18}$

$$\begin{aligned} &= \frac{0.1}{9.9} \\ 760 - P_s &= \frac{1}{99} P_s \\ P_s &= 752.4 \end{aligned}$$

110.  $\Delta H - \Delta U = \Delta n_g RT$   
 $= +\frac{1}{2} \times 8.314 \times 298$   
 $= +1238.78 \text{ J mol}^{-1}$



## Part-C

### Mathematics

111. Since  $\tan 30^\circ$  and  $\tan 15^\circ$  are roots of equation  $x^2 + px + q = 0$ .

$\therefore \tan 30^\circ + \tan 15^\circ = -p$   
 and  $\tan 30^\circ \tan 15^\circ = q$

Therefore

$$\begin{aligned} 2 + q - p &= 2 + \tan 30^\circ \tan 15^\circ \\ &\quad + (\tan 30^\circ + \tan 15^\circ) \\ &= 2 + \tan 30^\circ \tan 15^\circ \\ &\quad + 1 - \tan 30^\circ \tan 15^\circ \\ &\quad \left( \because \tan 45^\circ = \frac{\tan 30^\circ + \tan 15^\circ}{1 - \tan 30^\circ \tan 15^\circ} \right) \end{aligned}$$

$\Rightarrow 2 + q - p = 3$

112. Let  $I = \int_3^6 \frac{\sqrt{x}}{\sqrt{9-x} + \sqrt{x}} dx$  ... (i)

$$= \int_3^6 \frac{\sqrt{9-x}}{\sqrt{9-9+x} + \sqrt{9-x}} dx$$

$\Rightarrow I = \int_3^6 \frac{\sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}} dx$  ... (ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_3^6 \frac{\sqrt{x} + \sqrt{9-x}}{\sqrt{x} + \sqrt{9-x}} dx$$

$$= \int_3^6 1 dx = [x]_3^6$$

$$= 6 - 3 = 3$$

$\Rightarrow I = \frac{3}{2}$

113. Let  $W = \{\text{CAT}, \text{TOY}, \text{YOU}, \dots\}$

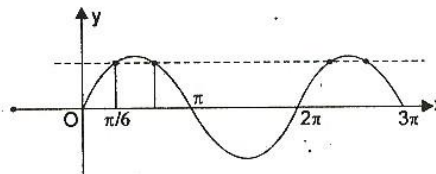
Clearly  $R$  is reflexive and symmetric but not transitive.

Since  $\text{CAT}R\text{TOY}$ ,  $\text{TOY}R\text{YOU} \not\Rightarrow \text{CAT}R\text{YOU}$

114. Since  $2 \sin^2 x + 5 \sin x - 3 = 0$

$\Rightarrow (2 \sin x - 1)(\sin x + 3) = 0$

$\Rightarrow \sin x = 1/2 \quad (\because \sin x \neq -3)$



From figure it is clear that the number of solutions of given equation in  $[0, 3\pi]$  is 4.

115.  $\because A^2 - B^2 = (A - B)(A + B)$

$\Rightarrow A^2 - B^2 = A^2 - B^2 + AB - BA$

$\Rightarrow AB = BA$

116.  $\sum_{k=1}^{10} \left( \sin \frac{2k\pi}{11} + i \cos \frac{2k\pi}{11} \right)$

$$= i \sum_{k=1}^{10} \left( \cos \frac{2k\pi}{11} - i \sin \frac{2k\pi}{11} \right)$$

$$= i \sum_{k=1}^{10} \left( e^{-\frac{2k\pi}{11}} \right)$$

$$= i \left\{ \sum_{k=0}^{10} \left( e^{-\frac{2k\pi}{11}} \right) - 1 \right\}$$

$$117. \because (\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$$

$$\Rightarrow (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{b} \cdot \vec{c}) \vec{a} = (\vec{a} \cdot \vec{c}) \vec{b} - (\vec{a} \cdot \vec{b}) \vec{c}$$

$$\Rightarrow (\vec{b} \cdot \vec{c}) \vec{a} = (\vec{a} \cdot \vec{b}) \vec{c}$$

$\Rightarrow \vec{a}$  is parallel to  $\vec{c}$ .

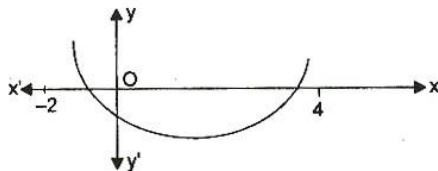
118. Since both roots of equation  $x^2 - 2mx + m^2 - 1 = 0$  are greater than  $-2$  but less than  $4$ .

$$\therefore D \geq 0$$

$$\Rightarrow 4m^2 - 4m^2 + 4 \geq 0 \Rightarrow m \in R \quad \dots(i)$$

$$\text{and } -2 < \frac{-b}{2a} < 4 \Rightarrow -2 < \left(\frac{2m}{2 \cdot 1}\right) < 4$$

$$\Rightarrow -2 < m < 4 \quad \dots(ii)$$



$$\text{Also } f(4) > 0 \Rightarrow 16 - 8m + m^2 - 1 > 0$$

$$\Rightarrow m^2 - 8m + 15 > 0$$

$$\Rightarrow (m - 3)(m - 5) > 0$$

$$\Rightarrow -\infty < m < 3 \text{ and } 5 < m < \infty \quad \dots(iii)$$

$$\text{Also } f(-2) > 0 \Rightarrow 4 + 4m + m^2 - 1 > 0$$

$$\Rightarrow m^2 + 4m + 3 > 0$$

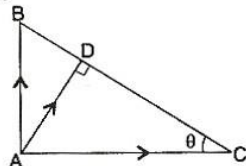
$$\Rightarrow (m + 3)(m + 1) > 0$$

$$\Rightarrow -\infty < m < -3 \text{ and } -1 < m < \infty \quad \dots(iv)$$

$\therefore$  From (i), (ii), (iii) and (iv)  
 $m$  lies between  $-1$  and  $3$ .

119. Let  $|\vec{BC}| = l$

$\therefore$  In  $\triangle ABC$ ,



$$l = \sqrt{AB^2 + AC^2}$$

$$\text{and } \tan \theta = \frac{AB}{AC}$$

$$\Rightarrow \sin \theta = \frac{AB}{l} \text{ and } \cos \theta = \frac{AC}{l}$$

$\therefore$  Resultant vector

$$= \frac{1}{AB} \hat{i} + \frac{1}{AC} \hat{j} = \left( \frac{1}{l \sin \theta} \hat{i} + \frac{1}{l \cos \theta} \hat{j} \right)$$

$$= k \vec{AD}$$

Now,  $AD = AC \sin \theta = l \sin \theta \cos \theta$

$$= \frac{AB \cdot AC}{l}$$

Magnitude of resultant vector

$$= \sqrt{\frac{1}{l^2} \left( \frac{1}{\sin^2 \theta} + \frac{1}{\cos^2 \theta} \right)}$$

$$= \frac{l}{(AB)(AC)} = \frac{1}{AD}$$

120. Since variance is independent of change in origin. Hence variance of observations  $101, 102, \dots, 200$  is same as variance of  $151, 152, \dots, 250$

$$\therefore V_A = V_B$$

$$\Rightarrow \frac{V_A}{V_B} = 1$$

121. Let  $I = \int_{-3\pi/2}^{-\pi/2} [(x + \pi)^3 + \cos^2(x + 3\pi)] dx \quad \dots(i)$

$$\text{and } I = \int_{-3\pi/2}^{-\pi/2} \left[ \left( -\frac{\pi}{2} - \frac{3\pi}{2} - x + \pi \right)^3 + \cos^2 \left( -\frac{\pi}{2} - \frac{3\pi}{2} - x + 3\pi \right) \right] dx$$

$$\Rightarrow I = \int_{-3\pi/2}^{-\pi/2} [-(x + \pi)^3 + \cos^2(\pi - x)] dx \quad \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2I = \int_{-3\pi/2}^{-\pi/2} 2 \cos^2 x dx$$

$$= \int_{-3\pi/2}^{-\pi/2} (1 + \cos 2x) dx$$

$$= \left[ x + \frac{\sin 2x}{2} \right]_{-3\pi/2}^{-\pi/2}$$

$$= -\frac{\pi}{2} + \frac{3\pi}{2} = \pi$$

$$\Rightarrow I = \frac{\pi}{2}$$

122. Given that  $2ae = 6$  and  $2b = 8$

$$\Rightarrow ae = 3 \text{ and } b = 4$$

$$\Rightarrow \frac{ae}{b} = \frac{3}{4}$$

$$\Rightarrow e^2 = \frac{9}{16} \left( \frac{b^2}{a^2} \right)$$

We know that  $\frac{b^2}{a^2} = 1 - e^2$



$$\Rightarrow e^2 = \frac{9}{16} (1 - e^2)$$

$$\Rightarrow \left(\frac{16+9}{9}\right) e^2 = 1 \Rightarrow e = \frac{3}{5}$$

123. The given equation of parabola is

$$y = \frac{a^3 x^2}{3} + \frac{a^2 x}{2} - 2a$$

$$\Rightarrow y + 2a = \frac{a^3}{3} \left[ x^2 + \frac{3}{2a} x \right]$$

$$\Rightarrow y + 2a = \frac{a^3}{3} \left[ x^2 + \frac{3}{2a} x + \frac{9}{16a^2} - \frac{9}{16a^2} \right]$$

$$\Rightarrow y + 2a = \frac{a^3}{3} \left[ x + \frac{3}{4a} \right]^2 - \frac{9}{16a^2} \times \frac{a^3}{3}$$

$$\Rightarrow y + 2a + \frac{3a}{16} = \frac{a^3}{3} \left( x + \frac{3}{4a} \right)^2$$

$$\Rightarrow \left( y + \frac{35a}{16} \right) = \frac{a^3}{3} \left( x + \frac{3}{4a} \right)^2$$

Thus the vertices of parabola is  $\left(-\frac{3}{4a}, -\frac{35a}{16}\right)$ .

Let  $h = -\frac{3}{4a}$

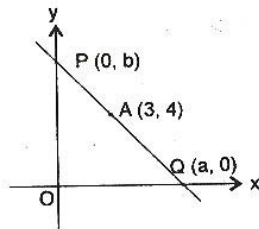
and  $k = -\frac{35a}{16}$

$\Rightarrow hk = \frac{105}{64}$

Thus the locus of vertices of a parabola is

$$xy = \frac{105}{64}$$

124. Since A is midpoint of line PQ



$$3 = \frac{a+0}{2} \Rightarrow a = 6$$

and  $4 = \frac{0+b}{2} \Rightarrow b = 8$

Thus the equation of line is

$$\frac{x}{6} + \frac{y}{8} = 1$$

$\Rightarrow 4x + 3y = 24$

125. Since position vectors of A, B, C are  $2\hat{i} - \hat{j} + \hat{k}$ ,  $\hat{i} - 3\hat{j} - 5\hat{k}$  and  $a\hat{i} - 3\hat{j} - \hat{k}$  respectively.

$\therefore \overrightarrow{AC} = (a\hat{i} - 3\hat{j} + \hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$

$$= (a-2)\hat{i} - 2\hat{j}$$

and  $\overrightarrow{BC} = (a\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} - 3\hat{j} - 5\hat{k})$   
 $= (a-1)\hat{i} + 6\hat{k}$

Since the  $\Delta ABC$  is right angled at C, then

$$\overrightarrow{AC} \cdot \overrightarrow{BC} = 0$$

$$\Rightarrow \{(a-2)\hat{i} - 2\hat{j}\} \cdot \{(a-1)\hat{i} + 6\hat{k}\} = 0$$

$$\Rightarrow (a-2)(a-1) = 0$$

$$\Rightarrow a = 1 \text{ and } 2$$

126. Let  $I = \int_0^{\pi} x f(\sin x) dx$  ... (i)

$$\Rightarrow I = \int_0^{\pi} (\pi - x) f(\sin(\pi - x)) dx$$

$$\Rightarrow I = \int_0^{\pi} (\pi - x) f(\sin x) dx$$
 ... (ii)

On adding Eqs. (i) and (ii), we get

$$2I = \int_0^{\pi} \pi f(\sin x) dx$$

$$\Rightarrow I = \frac{\pi}{2} \int_0^{\pi} f(\sin x) dx$$

$$\Rightarrow I = \pi \int_0^{\pi/2} f(\sin x) dx$$

Let  $\frac{\pi}{2} - x = t \Rightarrow x = \frac{\pi}{2} - t$

$$\therefore I = \frac{\pi}{2} \int_{-\pi/2}^{\pi/2} f(\cos t) dt$$

$$= \frac{\pi}{2} \int_{-\pi/2}^{\pi/2} f(\cos x) dx$$

$$= \pi \int_0^{\pi/2} f(\cos x) dx$$

( $\because f(\cos x)$  is an even function)

127. The equations of given lines are

$$x = ay + b, z = cy + d$$

and  $x = a'y + b', z = c'y + d'$

These equations can be rewritten as

$$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$$

and  $\frac{x-b'}{a'} = \frac{y-0}{1} = \frac{z-d'}{c'}$

These lines will perpendicular, if

$$aa' + cc' + 1 = 0$$

128. Total number of ways

$$= {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$$

$$= 10 + 45 + 120 + 210$$

$$= 385$$

129.  $\therefore (1-ax)^{-1} (1-bx)^{-1}$

$$= (1+ax+a^2x^2+\dots)(1+bx+b^2x^2+\dots)$$

Hence  $a_n =$  coefficient of  $x^n$  in  $(1-ax)^{-1} (1-bx)^{-1}$

$$= a^0b^n + ab^{n-1} + \dots + a^nb^0$$

$$= a^0b^n \left( \frac{\left(\frac{a}{b}\right)^{n+1} - 1}{\frac{a}{b} - 1} \right)$$

$$= \frac{b^n (a^{n+1} - b^{n+1})}{a-b} \cdot \frac{b}{b^{n+1}} = \frac{a^{n+1} - b^{n+1}}{a-b}$$

130.  $\therefore (1-y)^m (1+y)^n = ({}^mC_0 - {}^mC_1y + {}^mC_2y^2 - \dots)$   
 $({}^nC_0 + {}^nC_1y + {}^nC_2y^2 + \dots)$

$\therefore a_1 =$  coefficient of  $y$  in  $(1-y)^m (1+y)^n$

$$= {}^nC_1 - {}^mC_1 = 10$$

$$\Rightarrow n - m = 10$$

$$\Rightarrow n = m + 10 \quad \dots(i)$$

and  $a_2 =$  coefficient of  $y^2$  in  $(1-y)^m (1+y)^n$

$$= {}^nC_2 - {}^mC_1 \cdot {}^nC_1 + {}^mC_2$$

$$\therefore {}^nC_2 - {}^mC_1 \cdot {}^nC_1 + {}^mC_2 = 10$$

$$\Rightarrow \frac{n(n-1)}{2} + \frac{m(m-1)}{2} - mn = 10$$

$$\Rightarrow \frac{(10+m)(9+m)}{2} + \frac{m(m-1)}{2}$$

$$-m(10+m) = 10$$

(using Eq. (i))

$$\Rightarrow 45 - 5m + \frac{9m}{2} - \frac{m^2}{2} + \frac{m^2}{2} - \frac{m}{2} = 10$$

$$\Rightarrow 45 - m = 10 \Rightarrow m = 35$$

$$\therefore n = 45 \quad (\text{using Eq. (i)})$$

$$\therefore (m, n) = (35, 45)$$

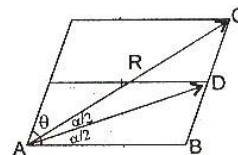
131. Let the magnitude of two velocities is  $u$ .

We know that

$$R = \sqrt{u^2 + u^2 + 2u^2 \cos \theta}$$

$$R = \sqrt{2u^2 (1 + \cos \theta)}$$

$$R = 2u \cos \frac{\theta}{2}$$



We know that by angle bisector theorem

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \frac{u}{2u \cos \frac{\theta}{2}} = 1$$

$$\Rightarrow \cos \frac{\theta}{2} = \frac{1}{2} = \cos \frac{\pi}{3}$$

$$\Rightarrow \theta = 120^\circ$$

132. Required probability =  $P(X=0) + P(X=1)$

$$= \frac{e^{-5}}{0!} 5^0 + \frac{e^{-5}}{1!} 5^1$$

$$= e^{-5} + 5e^{-5}$$

$$= \frac{6}{e^5}$$

133. Let the body is at a height  $h_1$  at a time  $t$  and is at height  $h$  at a time  $(t-4)$  from above.

$$\therefore h_1 - h = 400$$

$$\frac{1}{2}gt^2 - \frac{1}{2}g(t-4)^2 = 400$$

$$\Rightarrow t^2 - (t-4)^2 = 80$$

$$\Rightarrow (2t-4)4 = 80$$

$$\Rightarrow t = 12 \text{ seconds}$$

$$\therefore h = \frac{1}{2}g(t-4)^2$$

$$= 320 \text{ m}$$

Hence total distance of point  $P$  from the point from where body began to fall

$$= 320 + 400$$

$$= 720 \text{ m}$$

134.  $\therefore f(x) = \frac{x}{1+|x|}$

Let  $f(x) = \frac{g(x)}{h(x)} = \frac{x}{1+|x|}$

It is clear that  $g(x) = x$  and  $h(x) = 1+|x|$  are differentiable on  $(-\infty, \infty)$  and  $(-\infty, 0) \cup (0, \infty)$  respectively.

Thus  $f(x)$  is differentiable on  $(-\infty, 0) \cup (0, \infty)$ .

For  $x=0$

$$\lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{\frac{x}{1+|x|} - 0}{x} = \lim_{x \rightarrow 0} \frac{1}{1+|x|} = 1$$

Thus  $f(x)$  is differentiable on  $(-\infty, \infty)$ .

$$135. \therefore A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

$$\text{Now } AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \\ = \begin{bmatrix} a & 2b \\ 3a & 4b \end{bmatrix}$$

$$\text{and } BA = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \\ = \begin{bmatrix} a & 2a \\ 3b & 4b \end{bmatrix}$$

$$\text{If } AB = BA \Rightarrow a = b$$

Hence  $AB = BA$  is possible for infinitely many  $B$ 's.

$$136. \therefore \frac{a_1 + a_2 + \dots + a_p}{a_1 + a_2 + \dots + a_q} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{\frac{p}{2}[2a_1 + (p-1)d]}{\frac{q}{2}[2a_1 + (q-1)d]} = \frac{p^2}{q^2}$$

$$\Rightarrow \frac{(2a_1 - d) + pd}{(2a_1 - d) + qd} = \frac{p}{q}$$

$$\Rightarrow (2a_1 - d)(p - q) = 0$$

$$\Rightarrow a_1 = \frac{d}{2}$$

$$\text{Now } \frac{a_6}{a_{21}} = \frac{a_1 + 5d}{a_1 + 20d} = \frac{\frac{d}{2} + 5d}{\frac{d}{2} + 20d}$$

$$= \frac{11d}{41d} = \frac{11}{41}$$

$$137. \therefore f(x) = \frac{x}{2} + \frac{2}{x}$$

$$\therefore f'(x) = \frac{1}{2} - \frac{2}{x^2}$$

Put  $f'(x) = 0$  for maxima or minima

$$\frac{1}{2} - \frac{2}{x^2} = 0$$

$$\Rightarrow x^2 = 4 \Rightarrow x = \pm 2$$

$$\text{Now } f''(x) = \frac{4}{x^3} \Rightarrow f''(2) = \frac{4}{8} = \frac{1}{2} > 0$$

$$\text{and } f''(-2) = -\frac{4}{8} = -\frac{1}{2} < 0$$

$\therefore f(x)$  is minimum at  $x = 2$

$$138. \therefore y = x^2 - 5x + 6$$

$$\therefore \frac{dy}{dx} = 2x - 5$$

$$\text{Now } m_1 = \left(\frac{dy}{dx}\right)_{(2,0)} = 4 - 5 = -1$$

$$\text{and } m_2 = \left(\frac{dy}{dx}\right)_{(3,0)} = 6 - 5 = 1$$

Since  $m_1 m_2 = -1$

$\therefore$  Angle between the tangents is  $\frac{\pi}{2}$

$$139. \therefore \frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$$

$$= 1 + \frac{10}{3\left(x^2 + 3x + \frac{7}{3}\right)}$$

$$= 1 + \frac{10}{3\left[\left(x + \frac{3}{2}\right)^2 + \frac{1}{12}\right]}$$

$\Rightarrow$  Maximum value of  $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$  occurs at

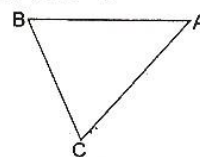
$$x = -\frac{3}{2}$$

$\therefore$  Maximum value of  $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$

$$= 1 + \frac{10}{3\left(\frac{1}{12}\right)}$$

$$= 1 + 40 = 41$$

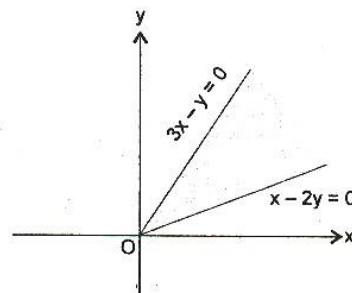
140. Given that  $AB = AC = x$



We know that area of isosceles triangle is maximum, if it is right angled triangle.

$\therefore$  Maximum area of triangle =  $\frac{1}{2}x^2$

141. The graph of equations  $x - 2y = 0$  and  $3x - y = 0$  is as shown in the figure. Since given point  $(a, a^2)$  lies in the shaded region.





Then  $a - 2a^2 < 0$  and  $3a - a^2 > 0$

$$\Rightarrow a \in (-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$$

and  $a \in (0, 3) \Rightarrow a \in \left(\frac{1}{2}, 3\right)$

142.  $\therefore x^m \cdot y^n = (x+y)^{m+n}$

Taking log on both sides, we get

$$m \log x + n \log y = (m+n) \log (x+y)$$

On differentiating with respect to  $x$ , we get

$$\begin{aligned} \frac{m}{x} + \frac{n}{y} \frac{dy}{dx} &= \frac{(m+n)}{(x+y)} \left(1 + \frac{dy}{dx}\right) \\ \Rightarrow \frac{dy}{dx} \left(\frac{m+n}{x+y} - \frac{n}{y}\right) &= \frac{m}{x} - \frac{m+n}{x+y} \\ \Rightarrow \frac{dy}{dx} \left(\frac{my + ny - nx - ny}{y(x+y)}\right) &= \frac{mx + my - mx - nx}{x(x+y)} \\ \Rightarrow \frac{dy}{dx} &= \frac{y}{x} \end{aligned}$$

143. The given equations of diameters are

$$3x - 4y - 7 = 0 \quad \dots(i)$$

and  $2x - 3y - 5 = 0 \quad \dots(ii)$

On solving Eqs. (i) and (ii), we get

$$x = 1 \text{ and } y = -1$$

$\therefore$  Centre of circle is  $(1, -1)$

Let  $r$  be the radius of circle, then

$$\pi r^2 = 49\pi$$

$$\Rightarrow r = 7 \text{ units}$$

$\therefore$  Equation of required circle is

$$(x-1)^2 + (y+1)^2 = 49$$

$$\Rightarrow x^2 + y^2 - 2x + 2y + 1 + 1 = 49$$

$$\Rightarrow x^2 + y^2 - 2x + 2y - 47 = 0$$

144. We know that the image  $(x, y, z)$  of a point  $(x_1, y_1, z_1)$  in a plane  $ax + by + cz + d = 0$  is given by

$$\begin{aligned} \frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} \\ = \frac{-2(ax_1 + by_1 + cz_1 + d)}{a^2 + b^2 + c^2} \end{aligned}$$

Thus image of point  $(-1, 3, 4)$  in a plane  $x - 2y = 0$  is given by

$$\begin{aligned} \frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} \\ = \frac{-2(1 \times (-1) + 3 \times (-2) + 0)}{1+4} \end{aligned}$$

$$\Rightarrow \frac{x+1}{1} = \frac{y-3}{-2} = \frac{z-4}{0} = \frac{-2(-7)}{5}$$

$$\Rightarrow x = \frac{14}{5} - 1 = \frac{9}{5}, y = -\frac{28}{5} + 3 = -\frac{13}{5}$$

and  $z = 4$

Thus image of point  $(-1, 3, 4)$  is  $\left(\frac{9}{5}, -\frac{13}{5}, 4\right)$ .

Hence no option is correct.

145. The given equation is  $Ax^2 + By^2 = 1$

On differentiating with respect to  $x$ , we get

$$2Ax + 2By \frac{dy}{dx} = 0 \quad \dots(i)$$

On again differentiating

$$2A + 2B \left\{ \left(\frac{dy}{dx}\right)^2 + y \frac{d^2y}{dx^2} \right\} = 0 \quad \dots(ii)$$

On solving Eqs. (i) and (ii), we get

$$y \frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 - \frac{y}{x} \frac{dy}{dx} = 0$$

This is the required differential equation whose order is 2 and degree is 1.

146.  $\therefore \int_1^a [x] f'(x) dx = \int_1^2 f'(x) dx$

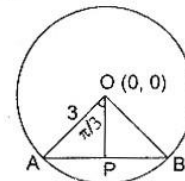
$$+ \int_2^3 2f'(x) dx + \dots + \int_{[a]}^a [a] f'(x) dx$$

$$= [a] f(x) - \{f(1) + f(2) + \dots + f([a])\}$$

147. Let the co-ordinates of a point  $P$  be  $(h, k)$  which is mid point of the chord  $AB$ .

$$\begin{aligned} OP &= \sqrt{(h-0)^2 + (k-0)^2} \\ &= \sqrt{h^2 + k^2} \end{aligned}$$

Now in  $\triangle OPA$ ,



$$\begin{aligned} \cos \frac{\pi}{3} &= \frac{OP}{OA} \\ \frac{1}{2} &= \frac{\sqrt{h^2 + k^2}}{3} \end{aligned}$$

$$\Rightarrow h^2 + k^2 = \left(\frac{3}{2}\right)^2$$

$$\Rightarrow h^2 + k^2 = \frac{9}{4}$$

Thus the required locus is  $x^2 + y^2 = \frac{9}{4}$